Problem Set 7

Practice: 9.5, 9.7, 9.11, 9.21, 10.5, 10.7

Due 10/27:

1. (2 points) Prove that in any simple graph, there are two vertices with the same degree.

2. (2 points) Find all non-isomorphic simple graphs on four vertices. How many non-isomorphic trees are there on four labelled vertices?

3. (2 points) If every vertex of some tournament $T$ of order $n$ has the same out-degree, then what is the average in-degree of the vertices?

4. (2 points) Let $G$ be a simple graph on $n$ vertices such that for any two vertices $v$ and $w$ of $G$, $\deg(v) + \deg(w) \geq n$. Prove that $G$ has a Hamiltonian cycle.

5. (2 points) Prove in any simple graph such that each vertex has degree at least 3 there is a cycle whose length is a power of 2.

6. (2 points) Show that in a tree, any two longest paths cross one another.

7. (2 points) Count the number of function $f : [n] \rightarrow [n]$ such that $|f^{-1}(\{1, 2, \ldots, i\})| \geq i$.

8. (2 points) Find an explicit formula for the number of trees on $n$ labeled nodes with exactly 4 leaves.