Practice Exam 2

1. For \( n > 1 \), find the maximum number of edges in a disconnected simple graph on \([n]\).

2. Let \( G \) be a simple graph with \( n \) vertices and \( k \) connected components. Prove that \( G \) is a forest if and only if \( G \) has \( n - k \) edges.

3. Find the number of spanning trees of the complete bipartite graph \( K_{n,m} \).

4. Let \( G \) be a planar graph such that every face is bordered by exactly 3 edges. What are the possible values for \( \chi(G) \)?

5. Find the number of perfect matchings for the cycle \( C_n \).