All questions will be worth 5 points. These questions are all from Munkres’ book. The answers here are brief!

(1) §10, no. 4
Solution: An increasing function on a closed interval has on a countable set of discontinuities. Let \( S_1 \) be the set of discontinuities of \( f \) and \( S_2 \) the set of discontinuities of \( g \). Then \( fg \) is continuous except at the sets \( S_1 \times [0, 1] \) and \( [0, 1] \times S_2 \). Each of these is a countable union of sets of measure zero, so \( fg \) is continuous off a set of measure zero, hence integrable.

(2) §10, no. 5
Solution: \( f \) vanishes off the countable set of rational points. Let \( x_n \to x \) be a sequence with an irrational limit. Then for given \( q \) and large \( n \) it cannot be the case that \( x_n \) is of the form \( p/q \) with \( p \) and \( q \) relatively prime. Thus it follows that \( \lim_{n \to \infty} f(x_n) = 0 \) so \( f \) is continuous at \( x \). Thus, \( f \) is continuous off the rationals and so is integrable on \( [0, 1] \).

(3) §11, no. 1
Take \( A \) the set of points in \([0, 1]^n\) with rational coefficients. It is countable, hence of measure zero. Its closure is the whole cube, so not of measure zero. Its boundary is \([0, 1]^n \setminus A \) so also not of measure zero since if it were then \([0, 1]^n \) would have measure zero, as the union of two such sets.

(4) §11, no. 2
It contains a ball of rectangle of positive volume, any covering by rectangles must have total volume larger than this.

(5) §11, no. 8
Since \( f \) vanishes off a closed set it is continuous on the complement (since it is locally constant, namely zero, near each point of the complement). Thus its set of discontinuities is contained in \( B \) hence has measure zero. Thus \( f \) is integrable. By Theorem 11.3 its integral vanishes.

(6) §11, no. 9
(a) If \( f(x) \geq 0 \) on \( Q \) then any lower partial sum with respect to a partition is non-negative, hence the integral itself is non-negative.
(b) By (a), \( \int_Q f \geq 0 \). So either it is positive or it vanishes. In the latter case it follows that all lower sums must vanish, since they are non-negative but smaller than than the integral. Thus \( f \) itself must vanish. Since \( f(x) > 0 \) this is not possible, so the integral is positive.