Solution Set #2, 18.440

Problem #5, p. 104: Let $A$ be the event that the first two balls are white, and let $B$ be the event that the last two are black. Then $P(A) = \frac{6 \times 5}{15 \times 14}$, $P(B|A) = \frac{9 \times 8}{15 \times 14}$, and $P(A \cap B) = P(B|A)P(A)$.

Problem #10, p. 104: Let $A$ be the event that the first card is a spade, and let $B$ be the event that the second and third cards are spades. Then $P(A \cap B) = \frac{13 \times 12 \times 11}{52 \times 51 \times 50}$, $P(A^C \cap B) = \frac{39 \times 38 \times 37}{52 \times 51 \times 50}$, and so $P(B) = P(A \cap B) + P(A^C \cap B) = \frac{1}{17}$. Hence, $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{11}{50}$.

Problem #13, p. 105: Let $C$ ($D$) be the event that a family owns a cat (a dog). The data given says that $P(C) = \frac{3}{10}$, $P(D) = \frac{9}{25}$, and $P(C|D) = \frac{11}{50}$. Thus, $P(C \cap D) = P(C|D)P(D) = \frac{11 \times 9}{50 \times 25} = \frac{99}{1250}$ and $P(D|C) = \frac{P(C \cap D)}{P(C)} = \frac{33}{125}$.

Problem #19, p. 106: $P(E_1) = 13 \frac{4 \times 48 \times \cdots \times 37}{52 \times 51 \times \cdots \times 40}$. $P(E_2|E_1) = 13 \frac{3 \times 36 \times \cdots \times 25}{39 \times 38 \times \cdots \times 27}$. $P(E_3|E_1 \cap E_2) = 13 \frac{2 \times 24 \times \cdots \times 13}{26 \times 25 \times \cdots \times 14}$. $P(E_4|E_1 \cap E_2 \cap E_3) = 1$. Finally, $P(E_1 \cap E_2 \cap E_3 \cap E_4) = P(E_1 \cap E_2 \cap E_3) = P(E_3|E_1 \cap E_2)P(E_1 \cap E_2) = P(E_3|E_1 \cap E_2)P(E_2|E_1)P(E_1)$. Of course, an easier way to do this calculation is to choose four 12 card hands from the 48 cards which are not aces, which can be done in $\binom{48}{12}$ ways, and then distribute the 4 aces, which can be done in $4!$ ways.

Problem #24, p. 107: Let $C$ be the event that the person is colorblind and $M$ that he is male. Then, $P(C|M) = \frac{1}{20}$ and $P(C|M^C) = \frac{1}{100}$. In addition, $P(M|C) = \frac{P(C|M)P(M)}{P(C|M)P(M) + P(C|M^C)(1 - P(M))}$. Finally, when there are equal numbers of men and women, $P(M) = \frac{1}{2}$, and when there are twice as many men as women, $P(M) = \frac{2}{3}$. 