Exercise 1. How many primitive roots are there for each of these numbers?
(a) 81  (b) 1250  (c) 36.

Exercise 2. Solve for $x \in \mathbb{Z}$ the following equation: $x^3 = 25 \cdot 64$.

Exercise 3. Prove that if $g$ and $g'$ are primitive roots modulo an odd prime $p$, then $gg'$ is NOT a primitive root modulo $p$.

Exercise 4. Let $p, q$ be primes such that $p \equiv 1 \pmod{4}$ and $q \equiv 3 \pmod{4}$. If the congruence equation $x^2 \equiv q \pmod{p}$ has no solutions, what can we say about the congruence equation $x^2 \equiv p \pmod{q}$?

Exercise 5. Consider the following matrix $A \in \text{Mat}_{3 \times 4}(\mathbb{Z})$
\[
\begin{pmatrix}
2 & 3 & 1 & 0 \\
0 & 1 & -1 & 3 \\
3 & 2 & 2 & 0
\end{pmatrix}.
\]
Find a Smith normal form $D$ for $A$, and matrices $U \in GL_3(\mathbb{Z})$ and $V \in GL_4(\mathbb{Z})$ such that $UAV = D$.

Exercise 6. Prove that the following equation has no integer solutions:
\[3x^2 + 3y^2 - z^2 - w^2 = 0.\]