Esercizio 1. Find the multiplicative inverse of 22 modulo 135.

Esercizio 2. Compute the remainder of $3^{64}$ in the division by 67.

Esercizio 3. Solve the following system of congruence equations:

\[
\begin{cases}
x \equiv 1 \pmod{3} \\
x \equiv 2 \pmod{5} \\
x \equiv 3 \pmod{7}
\end{cases}
\]

Esercizio 4. Prove that the following identity holds for every $n \in \mathbb{N}$:

\[
\sum_{i=0}^{n} \binom{n}{i} = n2^{n-1}
\]

Esercizio 5. For every $k \geq 1$ and every odd prime $p$, find the number of solutions of the following congruence equation

\[x^3 - x^2 + x - 1 \equiv 0 \pmod{p^k}.
\]

Esercizio 6. Let $P(x) \in \mathbb{Z}/p\mathbb{Z}[x]$ be a polynomial of degree $d$. Prove that $P(x)$ has $d$ distinct roots in $\mathbb{Z}/p\mathbb{Z}$ if and only if $P(x)$ divides $x^p - x$, namely

\[x^p - x \equiv P(x)Q(x) \pmod{p},
\]

for some polynomial $Q(x) \in \mathbb{Z}/p\mathbb{Z}[x]$. 
