1. MINIMAL SURFACES

a) Demonstrate that the shape of a fluid volume $V$ with minimum surface area is a sphere.

b) Show that the surface with the minimum area bounding two parallel rings is a catenoid. At what inter-ring spacing do you expect the surface to break, forming two circular surfaces bounding each ring.

c) Calculate the shape of the drop that forms an axisymmetric body of revolution around a cylindrical wire. Assume that the drop is sufficiently small that gravity is negligible, and that its interface meets the wire at a contact angle $\theta$.

2. CAPILLARY RISE BETWEEN PLATES

A pair of vertical plates are placed in a fluid bath, the gap spacing being much less than the capillary length.

a) Deduce the rise height of the fluid between the plates as a function of the equilibrium contact angle and the interfacial tension.

b) Describe the dynamics in the viscous limit, where the inertial effects are negligible.

c) Describe the dynamics in the inertial limit, where viscous effects are negligible. For the sake of simplicity, consider the long-time limit, where added mass effects are negligible.

d) The plates are now arranged so as to form a wedge with a small opening angle $\alpha$. Demonstrate that the resulting meniscus bound inside the plates takes the form of a hyperbola. Define this hyperbola in terms of the system parameters.
3. SMALL HYDRAULIC JUMPS

Consider an axysymmetric interface \( z = h(r) \) of the form shown below.

![Diagram of an axysymmetric interface](image)

a.). Calculate the radial curvature force that acts on the free surface by integrating the curvature pressure over its surface.

b.) Calculate and interpret this force for the case of an abrupt jump of the form shown below.

![Diagram of an abrupt jump](image)

c.) When a vertical jet impinges on a horizontal rigid plate, the fluid spreads until a critical radius at which the fluid depth increases abruptly in the form of a hydraulic jump. In the absence of surface tension, the jump radius is limited by the hydrostatic pressure, which exerts a radially inward force on the jump. When does one expect surface tension to significantly influence the jump radius?
4. ROLLING DROPS

Consider a toroidal fluid drop with outer radius $R$ and inner radius $r$.

a.) Calculate the force that acts to collapse the torus into a sphere.

b.) At what speed must a toroidal drop roll in order to be stable? (Consider the limit of $R \gg r$).

c.) As we have seen in class, such tori may be generated by rolling water drops down an inclined hydrophobic plane. What limits the size of such toroidal drops in a laboratory setting? Rationalize both upper and lower bounds.