18.02 Goals

1. Fluency in vector and matrix operations, including vector proofs and the ability to translate back and forth among the various ways to describe geometric properties, namely, in pictures, in words, in vector notation, and in coordinate notation

2. A comprehensive understanding of the gradient, including its relationship to level curves (or surfaces), directional derivatives, and linear approximation

3. The ability to set up problems involving several dependent and independent variables and/or constraints and
   a) to change variables or make the best choice of variables
   b) to change point of view by choosing to fix some variables while varying others

4. Ability to differentiate vector and scalar fields, including the chain rule

5. Ability to find differentials, to evaluate single, double and triple integrals, and to change variables in integration

6. Ability to apply the fundamental theorem of calculus on curves, Green’s theorem, the divergence theorem, and Stokes’s theorem

7. Exposure to some examples of partial differential equations and the derivation of these partial differential equations using physical principles and the divergence theorem

Students are exposed to the following list of terms, concepts and theorems. The goal is that they recognize these terms and the ideas behind them and that they know at least one physical and/or geometric context in which each one arises.

**Terms:** dot products, cross products, determinants, matrices, level curves and surfaces, partial derivatives, directional derivatives, gradients, critical points, the chain rule, constraints, Lagrange multipliers, iterated integrals, rectangular coordinates, polar coordinates, cylindrical coordinates, spherical coordinates, vector fields, conservative (exact) fields, line integrals, potential functions, surface integrals, flux, divergence, curl, the fundamental theorem of calculus for line integrals, Green’s theorem, Stokes’s theorem, the Laplace operator, the heat/diffusion equation.
18.03 Goals

Students should strive for personal mastery over the following skills.

1. Model a simple system to obtain a first order ODE.

2. Solve a first order linear ODE by the method of integrating factors or variation of parameter.

3. Calculate with complex numbers and exponentials.

4. Solve a constant coefficient second order linear initial value problem with driving term exponential times polynomial. In case of exponential (or sinusoidal) signal, compute amplitude gain and phase shift.

5. Utilize Delta functions in a signal, compute the unit impulse response, and express the system response to a general signal by means of the convolution integral.


7. Use Laplace transform to describe growth/decay and oscillation of functions of time, for large time, and (using tables and partial fractions) to find the weight function and solve constant coefficient linear initial value problems.

8. Calculate eigenvalues, eigenvectors, and matrix exponentials, and use them to solve homogeneous first order linear systems; relate linear systems with higher-order ODEs.

9. Recreate the phase portrait of a two-dimensional linear autonomous system from trace and determinant.

10. Determine the qualitative behavior of an autonomous nonlinear two-dimensional system by means of nullclines and an analysis of behavior near critical points.
18.06 Goals

The goals for 18.06 are using matrices and also understanding them. Here are key computations and some of the ideas behind them:

1. Solving \( Ax = b \) for square systems by elimination (pivots, multipliers, back substitution, invertibility of \( A \), factorization into \( A = LU \)).

2. Complete solution to \( Ax = b \) for general \( A \) (column space containing \( b \), rank of \( A \), nullspace of \( A \) and special solutions to \( Ax = 0 \) from row reduced \( R \)).

3. Basis and dimension (bases for the four fundamental subspaces).

4. Least squares solutions (closest line by understanding projections).

5. Orthogonalization by Gram-Schmidt (factorization into \( A = QR \)).

6. Properties of determinants (leading to the cofactor formula and the sum over all \( n! \) permutations, applications to \( A^{-1} \) and volume).

7. Eigenvalues and eigenvectors (diagonalizing \( A \), computing powers \( A^k \) and matrix exponentials to solve difference and differential equations).

8. Symmetric matrices and positive definite matrices (real eigenvalues and orthogonal eigenvectors, tests for \( x'Ax > 0 \), applications).

9. Linear transformations and change of basis (connected to the Singular Value Decomposition—orthonormal bases that diagonalize \( A \)).