18.433 Combinatorial Optimization Due: Oct 21

Assignment 2: Flow Theory

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1. Let G be a directed graph with two special vertices s and t. Any two directed paths from s to t are called *vertex-disjoint* if they do not share any vertices other than s and t. Prove that the maximum number of directed vertex-disjoint paths from s to t is equal to the minimum number of vertices whose removal ensures that there are no directed paths from s to t.

2. Let G be a directed graph with capacities on its edges and two special vertices s and t. The capacity of a directed path from s to t is the smallest of the capacities of edges on the path. Give an efficient algorithm to find a path from s to t of maximum possible capacity.

3. We say that a cut is within k times the mincut if the number of edges in the cut is within k times the number of edges in a mincut. Suppose that k is half an integer, i.e. $2k$ is an integer. Then show that in any undirected graph, the number of cuts within k times the mincut is fewer than n^{2k}.

4. Let P range over the set of $s-t$ paths for two vertices s, t of a given graph. Let C range over cuts that separate s and t. Then show that

$$\max_P \min_{e \in P} c_e = \min_C \max_{e \in C} c_e.$$

   Here c_e is the capacity of edge e.

5. Suppose that the maximum flow algorithm at each step augments on an augmenting path that has the least number of *reverse* arcs, i.e., flow will be sent backwards on these arcs in the augmenting path. Give a bound on the maximum number of augmentations performed by the algorithm.