RANDOM PLANAR MATCHING
AND BIN PACKING

by

Peter Williston Shor
B.S., California Institute of Technology

SUBMITTED TO THE
DEPARTMENT OF MATHEMATICS
IN PARTIAL FULFILLMENT
OF THE REQUIREMENTS FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY

at the
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

September, 1985

Copyright © Peter W. Shor, 1985

The author hereby grants to M.I.T. permission to reproduce and to distribute
copies of this thesis document in whole or in part.

Signature of Author

Certified by

Accepted by

Accepted by

Professor G. Theodore Leighton
Thesis Supervisor

Professor W. V. R. Malkus
Chairman, Applied Math Committee

Professor Neumith Ankeny
Chairman, Departmental Graduate Committee
Department of Mathematics

ARCHIVES

NOV 17 1985
RANDOM PLANAR MATCHING
AND BIN PACKING

by

Peter Williston Shor

Submitted to the Department of Mathematics
on September 6, 1985 in partial fulfillment of the requirements for
the degree of Doctor of Philosophy in Mathematics.

ABSTRACT

This thesis solves several well-known random planar matching problems, and
then applies the results to determine the average case behavior of two of the
most commonly used on-line algorithms for bin packing. In the planar matching
problems two kinds of points are distributed randomly in a unit square. We
investigate the length of the edges in an optimal matching between the two
kinds of points. Planar matching problems have arisen in several diverse areas
of mathematics, including probability, VLSI layout, and analysis of algorithms.
We define four planar matching problems: average edge length matching, right-
ward matching, up-right matching and maximum edge length matching. These
problems differ in the constraints we impose on the matchings and in the mea-
sure with respect to which the matching is optimized. We find new tight bounds
for up-right and maximum edge length matching.

We then use these matching problem to analyze the behavior of several on-line
bin packing algorithms. We find tight bounds on the expected wasted space in
the Best Fit algorithm, and bounds differing by a small factor for the expected
wasted space in the First Fit algorithm. We also provide a new lower bound for
the performance on any on-line algorithm. The performance of Best Fit is very
close to this theoretical lower bound.
Acknowledgements

It would be impossible to thank everyone who contributed to this thesis. There are many people to whom I am deeply indebted, and I will attempt to mention all of them.

I would like to thank all the people with whom I discussed bin packing and planar matching. These people include Noga Alon, Jon Bentley, Mark Halpern, David Johnson, Jeff Kahn, Tom Leighton, Charles Leiserson, Michael Luby, Daniel Kleitman, Cathy McGeoch, Lyle McGeoch and Gary Miller. Many ideas came out of these conversations.

I would like to thank Cathy McGeogh and Jon Bertley for sharing with me the results of their experimental studies of bin packing.

I would like to thank Jeff Kahn for suggesting a counterexample which led me to discover the lower bound on up-tight matching.

I would like to thank Noga Alon for noticing that the trees in Lemma 2.1.1 were really martingales.

I would like to thank all the friends that I had during my stay at M.I.T. — folk dancers, mathematics graduate students, roommates, and others — for their support, sympathy and patience.

I would like to thank the institutions that supported me financially during my graduate work. These are the National Science Foundation, which supported me with a Graduate Fellowship; the Air Force, which supported me through Air Force Grant OSR-82-0326; and the Mathematics Department, which supported me with a fellowship and with teaching assistantships.

I would like to thank the members of my thesis committee, Daniel Kleitman, Tom Leighton, and Charles Leiserson, for their criticism of the manuscript of
this thesis.

I would also like to thank the former members of my thesis committee, Gary Miller and Michael Sipser.

I would like to thank my advisor, Tom Leighton, whose contributions to this work are too numerous to list. In particular, I would like to thank him for his acuity in suggesting which problems I should attack, for his insights into how to solve them, and for his advice on how to present the solutions clearly.

I would like to thank my parents, Will and Joby Shor, without whom this never would have been possible.

Finally, I would like to thank my grandmother, Dorothy Shor, in whose house at an early age I discovered several marvelous examples of three-dimensional bin packing.