Sample Test Problems. Test 2: 18.310, 2005

1: For a linear program

$$\text{maximize} \quad x_1 + x_2 + 2x_3 \qquad \text{subject to} \;\; x_1, x_2, x_3 \geq 0$$

and

$$\begin{array}{rrrcr} & -x_2 & +x_3 & \leq & 6 \\ x_1 & +3x_2 & +2x_3 & \leq & 15 \\ x_1 & +4x_2 & & \leq & 10 \\ x_1 & & -2x_3 & \leq & 17 \end{array}$$

set up the initial tableau you would use for the simplex algorithm, including the slack variables. Suppose you decide to pivot on the variable x_3. Which row would you choose for the pivot? Why?

2:

WARNING: I am not sure this case is covered in the notes. It was gone over in class, and it will be in the solutions to the sample test problems, that I will post later this week. You should know it.

If you are performing an LP using the simplex method and you arrive at the following tableau:

x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	b
1	3	1	1	1	0	-3	0	-2	7
-1	2	-4	0	-2	1	-2	0	-1	2
2	1	2	0	-7	0	-3	1	-4	3
-4	-2	-5	0	-3	0	1	0	-5	-53

What are the values of the variables and the objective function at the vertex corresponding to this tableau? Can you do a pivot? If yes, which variable and which row would you pivot on. If not, what does this mean in terms of the solution to the linear program.

3: Suppose you are performing a linear program using the simplex method and you arrive at the following tableau:

x_1	x_2	x_3	x_4	x_5	x_6	b
3	1	0	1	0	1	7
1	-1	1	0	0	0	5
2	1	0	2	1	0	4
-4	2	0	3	0	0	-25

Which variables can you pivot on? Choose the leftmost of these and perform the pivot step. Give the resulting tableau. How much does this pivot step increase the objective function by?

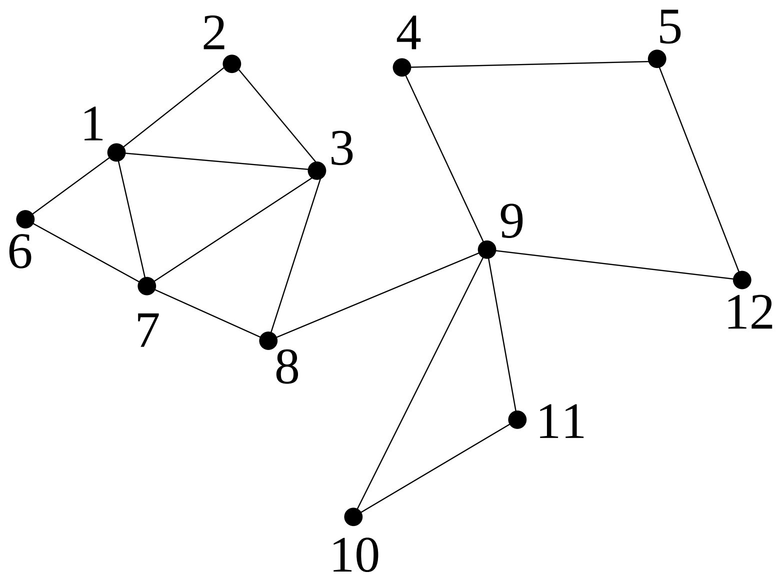
4: Given the following linear program. What is its dual?
WARNING: Prof. Kleitman's course notes define the dual wrong. This will not be an acceptable excuse if you make the same error on the exam. The OCR notes are correct, but confusing. I will try to link to or write some notes that are better later this week.

$$\text{maximize} \quad x_1 + x_2 + 2x_3 \qquad \text{subject to} \;\; x_1, x_2, x_3 \geq 0$$

and

$$\begin{array}{rrrcl} & -x_2 & +x_3 & \leq & 6 \\ x_1 & +3x_2 & +2x_3 & \leq & 15 \\ x_1 & +4x_2 & & \leq & 10 \\ x_1 & & -2x_3 & \leq & 17 \end{array}$$

5: Find depth-first-search and breadth-first-search trees for the following graph, starting at node 1.



6: You have two sequences f_0, f_1, …, f_{2^n-1} and g_0, g_1, …, g_{2^n-1}. You take the finite Fourier transform of each of these, to get a_0,a_1, …, a_{2^n-1} and b_0, b_1, …, b_{2^n-1}. If you know that for every i, one of f_i and g_i is 0, what does this tell you about a_i and b_i?

7. Suppose you have a polynomial

$$p(x) = a_0 + a_1 x + a_2 x^2 + \ldots + a_{2t+1} x^{2t+1}.$$

We let

$$p_{\text{even}} = a_0 + a_2 x + a_4 x^2 + a_6 x^3 + \ldots + a_{2t} x^t$$

and

$$p_{\text{odd}} = a_1 + a_3 x + a_5 x^2 + a_7 x^3 + \ldots + a_{2t+1} x^t.$$

Express $p(x^7)$ in terms of x, p_{even} and p_{odd} the way that you do in the recursion that gives rise to the FFT algorithm.

8: Given the recurrence $a_n = a_{n-1} + 6a_{n-2}$ starting with $a_0 = 2$, $a_1 = 5$, $a_2 = 17$, $a_3 = 47$, find a simple expression for a_k.

9: You have an $n \times 3$ strip of unit squares. Let T_n be the number of ways you can tile it with 1×1, 2×2, and 3×3 squares. Find a recurrence equation for T_n. Write down a polynomial $p(y)$ whose roots give numbers r_1, r_2 and r_3 so that

$$T_n = c_1 r_1^n + c_2 r_2^n + c_3 r_3^n$$

for some constants c_1, c_2, and c_3.