1: Write a few sentences about the topic of your term paper. [Note: the only way this question will affect the score for this homework will be if you don’t answer it.]

2: Find a breadth-first search tree starting at node 1 for the graph below.

3: Find a depth-first search tree starting at node 1 for the graph below. Show how the articulation point algorithm would use this depth-first search tree to compute the function $\text{Low}(v)$ for each vertex $v$. Find the biconnected components.

Problems 4a and 4b prove the theorem that, if a graph is biconnected, then for every pair of edges in the graph, there is a cycle containing both of them. Recall that a cycle is a set of distinct nodes $v_1, v_2, v_3, \ldots, v_k$ such that $(v_i, v_{i+1})$ is an edge, $1 \leq i \leq k - 1$ and $(v_k, v_1)$ is an edge. Recall that an articulation point of a connected graph $G$ is one whose deletion leaves more than one connected component. For problems 3 and 4, I will put up hints linked from the course webpage. For a challenge, try to solve these problems without hints, and then if you get stuck, look at the hints.

4a: Show that if some cycle in a graph contains edges $e$ and $f$, and some other cycle contains edges $f$ and $g$, then there is a cycle that contains edges $e$ and $g$.

4b: Possibly using problem 4a, show that for any pair of edges $e$ and $f$ in a connected graph with no articulation points, there is a cycle containing both of them.
5: Let $T_n$ be the number of ways of tiling a $3 \times n$ rectangle with $1 \times 1$ or $2 \times 2$ square tiles. This sequence starts off $T_1 = 1$, $T_2 = 3$, $T_3 = 5$, as shown below. Find a linear recurrence relation for $T_n$. Use this to find a simple expression for the generating function

$$g(x) = \sum_{n=0}^{\infty} T_n x^n.$$ 

Finally, express $T_n$ in the form

$$T_n = \sum_i c_i \alpha_i^n$$

for some real numbers $c_i$ and $\alpha_i$. 

![Image of tiling examples](image-url)