1: In Batcher’s algorithm, how many “rounds” of compare-and-switch operations (not using the same key twice in one round) are required to sort $2^k$ keys?

2: Set up a Batcher’s algorithm spreadsheet for 32 keys, so if you input 32 numbers on the left, they come out sorted on the right.
   How many comparisons do you need?
   How many do you need, in the worst case, for merge sorting and 32 keys?

3: Consider the sample corresponding to three tosses of a fair dice. These are three numbers, each of which has a probability $1/6$ of being each of the integers between 1 and 6. We then have a uniform sample space containing 216 elements, each of which has probability $1/216$. Now, condition on the sum of all three tosses being 12. We obtain a new uniform sample space, with fewer points.
   Compute the following quantities. Include a few sentences on how you did it (by all means use programs or spreadsheets if you find it easier, but say what you did).

3.1 What is size of the new sample space? What is the probability of the sum being 12 if three fair dice are tossed?

3.2 What is the expected value of the first toss (conditioned on the sum being 12)?

3.3 What is the most likely outcome of the first toss (conditioned on the sum being 12)?

3.4 What is the variance of the first toss (conditioned on the sum being 12)?

3.5 What is the covariance between the first and second tosses (conditioned on the sum being 12)?

3.6 Are the random variables corresponding to the first and second toss independent (conditioned on the sum being 12)?