1: This is problem 6.1 in Nielsen and Chuang. Suppose that $x_1, \ldots, x_N$ is a database of numbers held in memory, as in Section 6.5. Show that only $O(\log(N)\sqrt{N})$ accesses to the memory are required on a quantum computer, in order to find the smallest element on the list, with probability at least one-half.

2: Suppose that you have a qubit and want to perform the POVM with elements

$$
\begin{pmatrix}
0 & 1/2 & 1/4 \\
1/2 & 1/4 & 1/8 \\
1/4 & 1/8 & 1/8
\end{pmatrix} \begin{pmatrix}
1/2 & -1/4 \\
-1/4 & 1/8 \\
0 & 3/4
\end{pmatrix} \begin{pmatrix}
0 & 0 \\
0 & 0
\end{pmatrix}
$$

You can achieve the same probabilities of outcomes, for any given input state $|\psi\rangle$, by adding a qubit in state $|0\rangle$, applying a two-qubit unitary transformation $U$, and then applying the projective measurement in the computational basis. (And if you wish, you can duplicate this POVM exactly by subsequently preparing a qubit with the correct outcome.) Give a unitary transformation $U$ that will achieve this result.