

ERRATA AND ADDENDA

to

Enumerative Combinatorics, volume 1, second printing

by

Richard P. Stanley

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- p. 6, line 5–. Change compositon to composition.
- p. 11, Example 1.1.16, line 5. Change Y_i to Y_j (twice) and X_i to X_j .
- p. 16, line 3–. Change x_i to y_i .
- p. 19, line 19–. Change $[k]$ to $[n-k]$. This correction needs to be made in the hardcover but not the paperback edition.
- p. 19, line 4–. This should say $0 \leq a_i \leq x + n - i - 1$. It is correct in the hardcover edition and incorrect in the paperback edition.
- p. 19, line 7–. Change sufficies to suffices.
- p. 20, line 7–. It would be more accurate to replace “The proof of Proposition 1.3.7” with “The third proof of Proposition 1.3.4”.
- p. 24, Proposition 1.3.14, part 3, line 2. Replace “ k vertices” with “ $k-1$ vertices”. One does not need the bijection $\pi \rightarrow T(\pi)$ to see this. Any binary tree with k endpoints has $k-1$ vertices with two successors.
- p. 27, line 7. Add “let” after “Now”.
- pp. 30 and 31, Figures 1-6 and 1-7. The shading of these figures that appeared in the original printing was omitted from the second printing. In Figure 1-6, the boxes are shaded to denote the Young diagrams of the partitions \emptyset , (1) , (2) , $(1, 1)$, (3) , $(2, 1)$, $(3, 1)$, $(2, 2)$, $(3, 2)$, $(3, 3)$ in that order. In Figure 1-7, the boxes should be shaded above the lattice path L so that the shaded boxes form the Young diagram of the partition $(4, 3, 1)$.
- p. 33, Twelfefold Way, entries 7 and 10. The sum for entry 7 should begin $S(n, 0)$. Similarly the sum for entry 10 should begin $p_0(n)$. These terms are only relevant when $n = 0$ and yield the correct values $S(0, 0) = p_0(0) = 1$.
- p. 34, line 9–. Change (24a) to (24b).

- p. 42, lines 2–3. Further surveys of estimating the solution to an enumeration problem are A. M. Odlyzko, in *Handbook of Combinatorics*, vol. 1, Elsevier, Amsterdam, 1995, pp. 1063–1069, and the book P. Flajolet and R. Sedgewick, *Analytic Combinatorics*, to appear. Excerpts from this latter book are available at

<http://algo.inria.fr/flajolet/Publications/books.html>

- p. 45, Exercise 8(b). Add at the end of this exercise: “(Set $\binom{m}{i} = 0$ if $i < 0$.)”
- p. 47, Exercise 19(c), line 5. Change $f(n)$ to $f_k(n)$.
- p. 50, Exercise 1.40. The statement of this exercise is somewhat misleading, since the solution gives a formula for a_i not in terms of the f_n 's, but rather in terms of the g_n 's defined by $\log F(x) = \sum_{n \geq 1} g_n x^n$.
- p. 52, Exercise 2(a), line 3. Change $x + i + n + 1$ to $x + n + 1$.
- p. 55, Exercise 9(b). A simple combinatorial proof was given by the Cambridge Combinatorics and Coffee Club (December 1999).
- p. 56, Exercise 13, line 2. Change $b_1 < b_2 < \dots < b_m$ to b_1, b_2, \dots, b_m .
- p. 56, Figure 1-14. The next-to-last dot should be circled.
- p. 59, Exercise 26, line 3. Change $m_k(\mu)$ to $f_k(\mu)$.
- p. 59, Exercise 26, line 2–. Change function to functions.
- p. 59, Exercise 26. The following historical remarks concerning this exercise may be of interest. I discovered the result in 1972 and submitted it to the Problems and Solutions section of the *Amer. Math. Monthly*. It was rejected with the comment “A bit on the easy side, and using only a standard argument.” My guess is that the editors did not understand the actual statement and solution of the problem. I had mentioned the result to Daniel I. A. Cohen, who included the case $k = 1$ as Problem 75 of Chapter 3 in his book *Basic Techniques of Combinatorial Theory*, Wiley, New York, 1978. For this reason the case $k = 1$ is sometimes called “Stanley’s theorem.” An independent proof of the general case was given by Kirdar and Skyrme, as mentioned in the text (page 59). The generalization from $k = 1$ to arbitrary k was independently found by Paul Elder in 1984, as reported by R. Honsberger, *Mathematical Gems III*, Mathematical Association of America, 1985 (page 8). For this reason the general case is sometimes called “Elder’s theorem.” A further proof was given by A. H. M. Hoare, *Amer. Math. Monthly* **93** (1986), 475–476.
- p. 61, Exercise 33, line 1. Change $A(n, k)2^k$ to $A(n, k + 1)2^k$, and change k at the end of the line to $k - 1$.

- p. 62, line 8–. Change $\sum_{n \geq 0}$ to $\sum_{i \geq 0}$.
- p. 62, line 1–. Change \mathfrak{S}_{2n+1} to \mathfrak{S}_{2n-1} .
- p. 63, Exercise 45. A plausible explanation of the number 103,049 was found by David Hough and is discussed in R. Stanley, *Amer. Math. Monthly* **104** (1997), 344–350. A less convincing explanation of the number 310,952 appears in L. Habsieger, M. Kazarian, and S. Lando, *Amer. Math. Monthly* **105** (1998), 446. See also page 213 of *Enumerative Combinatorics*, vol. 2.
- p. 67, line 3. Insert a space before “has”.
- p. 70, equation (21). Change \mathbf{s}_j to \mathbf{s}_i (twice).
- p. 71, line 12–. Add the following sentence before this line (which is needed in the statement of Theorem 2.4.1).

Define the *rook polynomial* $r_B(x)$ of the board B by

$$r_B(x) = \sum_k r_k x^k.$$

- p. 72, line 10. Change “If” to “It”.
- p. 75, Theorem 2.4.4, lines 4–5. Change “only $s_1 \geq 0$ (i.e., $s_i < 0$ for $2 \leq i \leq t$) to $s_1 = 0$ and $s_i < 0$ for $2 \leq i \leq t$ ”.
- p. 80, lines 14– and 16–. Change “positive” to “nonnegative”.
- p. 80, lines 7– and 11–. Change τ' to $\tilde{\tau}$.
- p. 81, Figure 2-1. Change τ' to $\tilde{\tau}$.
- p. 82, §2.7, line 7. Change v_{i+i} to v_{i+1} .
- p. 84, lines 11–13. Change the sentence “Property (a) ... obtained from \mathbf{L} .” to “Property (a) follows since the triple (i, j, v) can be obtained from \mathbf{L}^* by the same rule as it can be obtained from \mathbf{L} .”
- p. 84, Example 2.7.2, line 7. Change α_i and δ_i to α_j and δ_j (twice).
- p. 85, Notes. Ferrers boards were first considered by D. Foata and M. P. Schützenberger, On the rook polynomials of Ferrers relations, *Colloquia Mathematica Societatis Janos Bolyai*, 4, Combinatorial Theory and Its Applications, vol. 2, (P. Erdős, A. Renyi, and V. Sós, eds.), North-Holland, Amsterdam, 1970, pp. 413–436.
- p. 88, equation (44). Change $1/n^5$ to $2/n^5$.

- p. 89, Exercise 11, line 3. Change “nents” to “nent”.
- p. 89, Exercise 11(b), line 5. Change G to \overline{G} .
- p. 92, line 5. Change A_T^a to V_T^a .
- p. 92, items **e,f**. Change $|T|$ to $|A_T|$.
- p. 93, Exercise 8(d). Change $f(n) + f(n + 1)$ to $f_2(n) + f_2(n + 1)$.
- p. 94, solution to Exercise 14, line 5. Change $k \leq -1$ to $k \geq -1$.
- p. 95, line 2. Change “regular” to “that every connected component is regular”.
- p. 96, line 3 after equation (1). Delete comma after C .
- p. 97, Example 3.1.1(d), line 3. Change [9] to 9.
- p. 111, Example 3.5.3, line 6. Change second *bacde* to *badce*.
- p. 111, Figure 3-24. The shading of this figure that appeared in the original printing was omitted from the second printing. The entire inside region should be shaded.
- p. 111, line 3-. Change “ m element” to “ m -element”.
- p. 112, lines 7- to 6-. Change $e(J(\mathbf{m}+\mathbf{n})) = e(\mathbf{m} + \mathbf{1} \times \mathbf{n} + \mathbf{1}) = \binom{m+n}{n}$ to $e(\mathbf{m}+\mathbf{n}) = \binom{m+n}{n}$.
- p. 117, line 13. Change $T < 1$ to $T < \hat{1}$.
- p. 117, line 4-. After \Leftrightarrow insert “ $f(0) = g(0)$ and”.
- p. 120, line 10. Insert Δ after “collection”.
- p. 122, Figure 3-29. Shading is missing from Γ_4 , Γ_5 , and Γ_6 . All two-dimensional regions (including the outside one) are 2-cells.
- p. 125, line 6. Change \mathbb{C} to K .
- p. 125, line 9-. Change \mathbb{C} to K .
- p. 125, line 8-. Change $y \leq 1$ to $y \leq \hat{1}$.
- p. 127, lines 2 and 6. Change L to L_n .
- p. 127, Example 3.10.3, line 2. Change “Note” to “For the purpose of this example, we say”. If one wants to retain the more standard convention that the empty set spans $\{0\}$, then we need to enlarge $L_n(q)$ by adding \emptyset below $\{0\}$.

- p. 128, line 14. The type of a set partition $\pi \in \Pi_n$ has not been defined, though the definition should be clear in analogy to the type of a permutation. Namely, define $\text{type}(\pi) = (a_1, \dots, a_n)$ if π has a_i blocks of size i .
- p. 128, line 16. Change $=$ to \cong .
- p. 131, line 6 of text. Change “rank i ” to “of rank i ”.
- p. 131, line 5–. Change $\sigma : P \rightarrow [n]$ to $\sigma : P \rightarrow \mathbf{n}$.
- p. 133, line 13–. Change $a_0 = \hat{0}, a_{s+1} = \hat{1}$ to $a_0 = 0, a_{s+1} = n$.
- p. 136, line 6. Change $(-1)^n \Delta Z(P, m-1)$ to $(-1)^{n-1} \Delta Z(P, m-1)$.
- p. 137, line 7. Change Q to $P - Q$.
- p. 142, line 14–. It should have been stated that $\text{card}(x, y)$ is short for $\text{card}([x, y])$.
- p. 152, reference 12, line 2. Change *functions* to *function*.
- p. 153, Exercise 1a, line 1. Change “operation” to “relation”.
- p. 155, line 4. Insert “irreducible” before “*connected*”. (An irreducible poset is one that cannot be written in a nontrivial way as a direct product.)
- p. 156, Exercise 15(d). This exercise was solved by J. D. Farley and R. Klippenstine, *Posets with the same number of order ideals of each cardinality, II*, preprint dated November 30, 2004.
- p. 157, Exercise 22(e). It was shown by J. Farley, *J. Combinatorial Theory (A)* **90** (2000), 123–147, that the only nondecreasing cover functions (with $f(0) \geq 1$) are $f(n) = k$ and $f(n) = n + k$ for $k \geq 1$. This confirms a conjecture of R. Stanley, *Fibonacci Quart.* **13** (1975), 215–232 (page 226).
- p. 160, line 1. Change second P to \bar{P} .
- p. 160, Exercise 32, line 4. Insert $\mu(x_k, \hat{1})$ after $\mu(x_{k-1}, x_k)$.
- p. 164, line 1. Change a_i to y (twice).
- p. 166, line 1–. Change X_1 to H_1 and X_ν to H_ν .
- p. 167, Exercise 59a. Change the last sentence to: Show that if $Z(P, m+1) = \sum_{i \geq 1} a_i \binom{m-1}{i}$, then $Z(Q_0, m+1) = 1 + \sum_{i \geq 1} a_i m^i$.
- p. 174, Exercise 81a, lines 3–4. Change “define f, g ” to “define g, h ”.

- p. 174, Exercise 81c, line 5. Change this line to

$$1 + t \sum_{n \geq 1} G_n(q, t) x^n / (\mathbf{n})! = \left[1 - t \sum_{n \geq 1} (1 - t)^{n-1} x^n / (\mathbf{n})! \right]^{-1}.$$

- p. 174, Exercise 81c, line 2-. Change $(1 - t)/(e^{x(t-1)} - 1)$ to $(1 - t)/(e^{x(t-1)} - t)$.
- p. 178, solution to Exercise 19a, lines 5–7. Interchange f_k and g_k .
- p. 183, solution to Exercise 30, line 5. Change Q to \bar{P} (under the summation sign).
- p. 184, solution to Exercise 32, line 3. Insert $\mu(x_k, \hat{1})$ after $\mu(x_{k-1}, x_k)$.
- p. 184, solution to Exercise 37b, line 1. Change “ $f(x, s) = x$ ” to “ $f(x, s) = \phi(x)$ (so $F(x, s) = x$)”.
- p. 187, line 2. Remove $\prod_{i=1}^k$.
- p. 187, line 3-. Change $x^{n-\dim W}$ to $x^{n-\dim W'}$.
- p. 188, Exercise 46, second solution, lines 1–2. Change $a \neq \{1\}$ to $\emptyset \neq a$.
- p. 191, Exercise 50, line 4. Under the third summation symbol, change $x \in P_j$ to $y \in P_j$.
- p. 191, Exercise 51, line 5. Change “voltage graphs” to “gain graphs”.
- p. 191, Exercise 51, line 7. A general reference for enumerative results on gain graphs is T. Zaslavsky, *J. Combin. Theory Ser. B* **64** (1995), 17–88.
- page 192, line 2-. This formula should be:

$$Z(P \oplus Q, m) = Z(P, m) + Z(Q, m) + \sum_{j=2}^{m-1} Z(P, j)Z(Q, m+1-j), \quad m \geq 2.$$

- p. 193, line 1. Change “chains” to “multichains”.
- p. 195, solution to Exercise 61b. Interchange \mathbf{p} and $p\mathbf{1}$ throughout.
- p. 196, line 16-. Change $x_{\rho(x_i)}$ to $t_{\rho(x_i)}$.
- p. 197, Exercise 69(d), lines 2- to 1-. Update this reference to *Discrete Math.* **79** (1990), 235–249.
- p. 200, Exercise 80, line 4. Change $6n+3$ to $6n + 3$, i.e., the 3 should not be italicized. Could this be the most nitpicking error of this errata?

- p. 206, first line of proof. Change $R(x)$ to $F(x)$.
- p. 216, line 5. Change 1.3.3 to 1.3.
- p. 224, line 3. Change “rank $d = \dim \mathcal{C}$ to “rank $d + 1$ where $d = \dim \mathcal{C}$ ”.
- p. 227, line 8. Change “ $a_i = [b_i]$, the least integer $\geq b_i$ ” to “ $a_i = [b_i - 1]$, where $[b_i]$ denotes the least integer $\geq b_i$ ”.
- p. 230, line 2 of Proof. Change $d - \dim \sigma$ to $d - \dim \sigma + 1$.
- p. 231, Lemma 4.6.17(i). As stated, the result is false. For instance, let $E = \mathbb{N}$ and $a_1 = -1$. Then $G(\lambda) = 1$ but $E(\lambda^{-1}) = \sum_{n \geq 0} \lambda^{-n}$. We need to assume also that $g(r) > 0$ for at least one $r > 0$. We claim that then $g(s) = 0$ for all $s < 0$, and hence (i) follows. Let $\alpha \in E$ satisfy $L(\alpha) = r > 0$, and suppose that there exists $\beta \in E$ with $L(\beta) = s < 0$. Then for all $t \in \mathbb{N}$ the vectors $ts\alpha + tr\beta$ are distinct elements of E , contradicting $g(0) < \infty$.
- p. 236, line 7. Change “union” to “intersection”.
- p. 237, line 8. Change $\text{den}(\gamma, t)$ to $\text{den}(\gamma/t)$.
- p. 244, Figure 4-13. Add an edge from 13 to 31.
- p. 246, lines 5–6. Change 7 to 6 (twice).
- p. 249, line 15–. Add at end of line: where $b(n) = \sum_{v \in \mathcal{B}_n} w(v)$.
- p. 253, line 4–. Change D_4 to D_3 .
- p. 256, line 2. Change $(j - 1, j)$ to $(j, j - 1)$.
- p. 257, line 19–. Change $f_s(n_s)$ to $f_k(n_k)$.
- p. 257, line 18–. Change $f_s(n_{s+1})$ to $f_k(n_{k+1})$.
- p. 260, Figure 4-42. Change the label on the edge from 00 to 01 from $F_1 * F_2$ to $F_1 * F_3$.
- p. 262, line 5–. The first published statement for the generating function for $F(x)$ appearing before equation (47) seems to be due to H. N. V. Temperley, *Phys. Rev. (2)* **103** (1956), 1–16.
- p. 262, line 2–. The result of Hickerson has now appeared in *J. Integer Sequences* (electronic) **2** (1999), Article 99.1.8,

<http://www.research.att.com/~njias/sequences/JIS/HICK2/chcp.html>.

- p. 266, Exercise 12. Change $\Phi\alpha = \mathbf{0}$ to $\Phi\alpha = \beta$, where β is a vector of linear polynomials $an + b$. Moreover, the final sentence should be “Show that \mathbb{N}^m can partitioned into finitely many regions such that on each region the number of solutions is a quasipolynomial in n for n sufficiently large.” It is possible that this result is already known.
- p. 271, Exercise 27(a), line 2. Change this line to

$$x_1 + x_2 + \cdots + x_r \leq 1, \quad y_1 + y_2 + \cdots + y_s \leq 1, \quad x_i \geq 0, \quad y_i \geq 0.$$

- p. 286, Exercise 23. Peter McNamara has pointed out a gap in the proof. Namely, from the fact that $P - M$ and $P - M'$ are disjoint unions of chains, it need not follow (when P has maximal chains of length one) that P is a disjoint union of chains, together with the stated relations $x < y$. To fix the proof, note that m is the largest power of x_0 that can appear in a monomial in $\bar{G}(P, \mathbf{x})$. Hence m is the largest power of *any* x_i that can appear in a monomial in $\bar{G}(P, \mathbf{x})$. Let A be an antichain of P . We can easily find a strict P -partition that is constant on A , so $\#A \leq m$. Hence the largest antichain of P has size m . By Dilworth’s theorem, P is a union of m chains. Each such chain intersects M and M' . It is now easy to see that if $P - M$ and $P - M'$ are disjoint unions of chains, then indeed P is a disjoint union of chains together with the stated relations $x < y$, and the proof proceeds as before.
- p. 293, line 9–. In the definition of a connected graph, it should also be specified that the empty graph is *not* connected.
- p. 295, Figure A-2. Interchange the labels 3 and 5 on the third tree.
- p. 308, Problem 7, line 3. Change $[n]$ to $[n - 1]$.
- p. 310, Problem 25, line 2. Change “occurences” to “occurrences” (twice).
- p. 310, Problem 25, line 3. Change “ t occurences of each a_{ij} ” to “ $2t$ occurrences of each a_{ij} ”.
- p. 314, Problem 6(c). A solution was found by Ethan Fenn (private communication, November, 2002). The rating should be changed to [3–] and the problem restated as follows.

Let $k \geq 3$, and let P_k denote the poset of all subsets of \mathbb{P} whose elements have sum divisible by k . Given $T \leq S$ in P_k , let

$$i_j = \#\{n \in T - S : n \equiv j \pmod{k}\}.$$

Clearly $\mu(S, T)$ depends only on the k -tuple $(i_0, i_1, \dots, i_{k-1})$, so write $\mu(i_0, \dots, i_{k-1})$ for $\mu(S, T)$. Show that

$$\begin{aligned} & \sum_{i_0, \dots, i_{k-1} \geq 0} \mu(i_0, \dots, i_{k-1}) \frac{x_0^{i_0} \cdots x_{k-1}^{i_{k-1}}}{i_0! \cdots i_{k-1}!} \\ &= k \left[\sum_{j=0}^{k-1} \exp(x_0 + \zeta^j x_1 + \zeta^{2j} x_2 + \cdots + \zeta^{(k-1)j} x_{k-1}) \right]^{-1}, \end{aligned}$$

where ζ is a primitive k th root of unity.

- p. 314, Problem 10, line 1. Delete the difficulty rating [2+] at the beginning of the line.
- p. 314, Problem 13, lines 2–3. Delete the difficulty rating [2+] at the beginning of these lines.
- p. 314, line 2–. Delete “(ii)”.
- p. 318, Problem 13. This should be rated [2+].
- p. 319, line 9–. This erratum is unnecessary and can be deleted.
- p. 320, line 2 (paperback edition only). Change $(2^{a_1} - 1) \cdots (2^{a_1} - 1)$ to $(2^{a_1-1} - 1) \cdots (2^{a_k-1} - 1)$.
- p. 320, lines 16–17. Delete this item.
- p. 321, item 3, line 2. Change $\mu(kn)$ to $\mu_S(kn)$.
- p. 321, item 6. The computation of $f(14) = 1338193159771$ is given by J. Heitzig and J. Reinhold, *Order* **17** (2000), 333–341. They also compute the number of *labelled* n -element posets for $n \leq 16$. According to Vledeta Jovović, as reported on the *Encyclopedia of Integer Sequences*, A000112, <http://www.research.att.com/~njas/sequences>, we have $f(15) = 68275077901156$ and $f(16) = 4483130665195087$.
- p. 321, line 7. Delete this entry (for p. 149, line 10).
- p. 322, lines 8– to 5–. The stated result is false. The hypothesis on L should be the following: L is an n -element lattice such that for all $\hat{0} < x \leq y$ in L , there exists $z \neq y$ such that $z \vee x = y$.
- p. 322, line 4–. Change ℓ^{k-1} to $k^{\ell-1}$ ($\ell \geq 2$).