

# ERRATA AND ADDENDA

to

*Enumerative Combinatorics*, volume 1, second printing

by

Richard P. Stanley

(version of 25 April 2008)

- p. 11, Example 1.1.16, line 5. Change  $Y_i$  to  $Y_j$  (twice) and  $X_i$  to  $X_j$ .
- p. 16, line 3–. Change  $x_i$  to  $y_i$ .
- p. 19, line 19–. Change  $[k]$  to  $[n-k]$ . This correction needs to be made in the hardcover but not the paperback edition.
- p. 19, line 4–. This should say  $0 \leq a_i \leq x + n - i - 1$ . It is correct in the hardcover edition and incorrect in the paperback edition.
- p. 19, line 7–. Change sufficies to suffices.
- p. 20, line 7–. It would be more accurate to replace “The proof of Proposition 1.3.7” with “The third proof of Proposition 1.3.4”.
- p. 24, Proposition 1.3.14, part 3, line 2. Replace “ $k$  vertices” with “ $k-1$  vertices”. One does not need the bijection  $\pi \rightarrow T(\pi)$  to see this. Any binary tree with  $k$  endpoints has  $k-1$  vertices with two successors.
- p. 27, line 7. Add “let” after “Now”.
- pp. 30 and 31, Figures 1-6 and 1-7. The shading of these figures that appeared in the original printing was omitted from the second printing. In Figure 1-6, the boxes are shaded to denote the Young diagrams of the partitions  $\emptyset$ , (1), (2), (1, 1), (3), (2, 1), (3, 1), (2, 2), (3, 2), (3, 3) in that order. In Figure 1-7, the boxes should be shaded above the lattice path  $L$  so that the shaded boxes form the Young diagram of the partition (4, 3, 1).
- p. 33, Twelfefold Way, entries 7 and 10. The sum for entry 7 should begin  $S(n, 0)$ . Similarly the sum for entry 10 should begin  $p_0(n)$ . These terms are only relevant when  $n = 0$  and yield the correct values  $S(0, 0) = p_0(0) = 1$ .
- p. 34, line 9–. Change (24a) to (24b).

- p. 42, lines 2–3. Further surveys of estimating the solution to an enumeration problem are A. M. Odlyzko, in *Handbook of Combinatorics*, vol. 1, Elsevier, Amsterdam, 1995, pp. 1063–1069, and the book P. Flajolet and R. Sedgewick, *Analytic Combinatorics*, to appear. Excerpts from this latter book are available at

<http://algo.inria.fr/flajolet/Publications/books.html>

- p. 45, Exercise 8(b). Add at the end of this exercise: “(Set  $\binom{m}{i} = 0$  if  $i < 0$ .)”
- p. 47, Exercise 19(c), line 5. Change  $f(n)$  to  $f_k(n)$ .
- p. 50, Exercise 1.40. The statement of this exercise is somewhat misleading, since the solution gives a formula for  $a_i$  not in terms of the  $f_n$ 's, but rather in terms of the  $g_n$ 's defined by  $\log F(x) = \sum_{n \geq 1} g_n x^n$ .
- p. 52, Exercise 2(a), line 3. Change  $x + i + n + 1$  to  $x + n + 1$ .
- p. 55, Exercise 9(b). A simple combinatorial proof was given by the Cambridge Combinatorics and Coffee Club (December 1999).
- p. 56, Exercise 13, line 2. Change  $b_1 < b_2 < \dots < b_m$  to  $b_1, b_2, \dots, b_m$ .
- p. 56, Figure 1-14. The next-to-last dot should be circled.
- p. 59, Exercise 26, line 3. Change  $m_k(\mu)$  to  $f_k(\mu)$ .
- p. 59, Exercise 26, line 2–. Change function to functions.
- p. 59, Exercise 26. The following historical remarks concerning this exercise may be of interest. I discovered the result in 1972 and submitted it to the Problems and Solutions section of the *Amer. Math. Monthly*. It was rejected with the comment “A bit on the easy side, and using only a standard argument.” My guess is that the editors did not understand the actual statement and solution of the problem. I had mentioned the result to Daniel I. A. Cohen, who included the case  $k = 1$  as Problem 75 of Chapter 3 in his book *Basic Techniques of Combinatorial Theory*, Wiley, New York, 1978. For this reason the case  $k = 1$  is sometimes called “Stanley’s theorem.” An independent proof of the general case was given by Kirdar and Skyrme, as mentioned in the text (page 59). The generalization from  $k = 1$  to arbitrary  $k$  was independently found by Paul Elder in 1984, as reported by R. Honsberger, *Mathematical Gems III*, Mathematical Association of America, 1985 (page 8). For this reason the general case is sometimes called “Elder’s theorem.” A further proof was given by A. H. M. Hoare, *Amer. Math. Monthly* **93** (1986), 475–476.
- p. 61, Exercise 33, line 1. Change  $A(n, k)2^k$  to  $A(n, k + 1)2^k$ , and change  $k$  at the end of the line to  $k - 1$ .

- p. 62, line 8–. Change  $\sum_{n \geq 0}$  to  $\sum_{i \geq 0}$ .
- p. 62, line 1–. Change  $\mathfrak{S}_{2n+1}$  to  $\mathfrak{S}_{2n-1}$ .
- p. 63, Exercise 45. A plausible explanation of the number 103,049 was found by David Hough and is discussed in R. Stanley, *Amer. Math. Monthly* **104** (1997), 344–350. A less convincing explanation of the number 310,952 appears in L. Habsieger, M. Kazarian, and S. Lando, *Amer. Math. Monthly* **105** (1998), 446. See also page 213 of *Enumerative Combinatorics*, vol. 2.
- p. 67, line 3. Insert a space before “has”.
- p. 70, equation (21). Change  $\mathbf{s}_j$  to  $\mathbf{s}_i$  (twice).
- p. 71, line 12–. Add the following sentence before this line (which is needed in the statement of Theorem 2.4.1).

Define the *rook polynomial*  $r_B(x)$  of the board  $B$  by

$$r_B(x) = \sum_k r_k x^k.$$

- p. 72, line 10. Change “If” to “It”.
- p. 75, Theorem 2.4.4, lines 4–5. Change “only  $s_1 \geq 0$  (i.e.,  $s_i < 0$  for  $2 \leq i \leq t$ ) to  $s_1 = 0$  and  $s_i < 0$  for  $2 \leq i \leq t$ ”.
- p. 80, lines 14– and 16–. Change “positive” to “nonnegative”.
- p. 80, lines 7– and 11–. Change  $\tau'$  to  $\tilde{\tau}$ .
- p. 81, Figure 2-1. Change  $\tau'$  to  $\tilde{\tau}$ .
- p. 82, §2.7, line 7. Change  $v_{i+i}$  to  $v_{i+1}$ .
- p. 84, lines 11–13. Change the sentence “Property (a) ... obtained from  $\mathbf{L}$ .” to “Property (a) follows since the triple  $(i, j, v)$  can be obtained from  $\mathbf{L}^*$  by the same rule as it can be obtained from  $\mathbf{L}$ .”
- p. 84, Example 2.7.2, line 7. Change  $\alpha_i$  and  $\delta_i$  to  $\alpha_j$  and  $\delta_j$  (twice).
- p. 85, Notes. Ferrers boards were first considered by D. Foata and M. P. Schützenberger, On the rook polynomials of Ferrers relations, *Colloquia Mathematica Societatis Janos Bolyai*, 4, Combinatorial Theory and Its Applications, vol. 2, (P. Erdős, A. Renyi, and V. Sós, eds.), North-Holland, Amsterdam, 1970, pp. 413–436.
- p. 88, equation (44). Change  $1/n^5$  to  $2/n^5$ .

- p. 89, Exercise 11, line 3. Change “nents” to “nent”.
- p. 89, Exercise 11(b), line 5. Change  $G$  to  $\overline{G}$ .
- p. 92, line 5. Change  $A_T^a$  to  $V_T^a$ .
- p. 92, items **e,f**. Change  $|T|$  to  $|A_T|$ .
- p. 93, Exercise 8(d). Change  $f(n) + f(n + 1)$  to  $f_2(n) + f_2(n + 1)$ .
- p. 94, solution to Exercise 14, line 5. Change  $k \leq -1$  to  $k \geq -1$ .
- p. 95, line 2. Change “regular” to “that every connected component is regular”.
- p. 96, line 3 after equation (1). Delete comma after  $C$ .
- p. 97, Example 3.1.1(d), line 3. Change [9] to 9.
- p. 111, Example 3.5.3, line 6. Change second *bacde* to *badce*.
- p. 111, Figure 3-24. The shading of this figure that appeared in the original printing was omitted from the second printing. The entire inside region should be shaded.
- p. 111, line 3–. Change “ $m$  element” to “ $m$ -element”.
- p. 112, lines 7– to 6–. Change  $e(J(\mathbf{m} + \mathbf{n})) = e(\mathbf{m} + \mathbf{1} \times \mathbf{n} + \mathbf{1}) = \binom{m+n}{n}$  to  $e(\mathbf{m} + \mathbf{n}) = \binom{m+n}{n}$ .
- p. 117, line 13. Change  $T < 1$  to  $T < \hat{1}$ .
- p. 117, line 4–. After  $\Leftrightarrow$  insert “ $f(0) = g(0)$  and”.
- p. 120, line 10. Insert  $\Delta$  after “collection”.
- p. 122, Figure 3-29. Shading is missing from  $\Gamma_4$ ,  $\Gamma_5$ , and  $\Gamma_6$ . All two-dimensional regions (including the outside one) are 2-cells.
- p. 125, line 8–. Change  $y \leq 1$  to  $y \leq \hat{1}$ .
- p. 127, lines 2 and 6. Change  $L$  to  $L_n$ .
- p. 127, Example 3.10.3, line 2. Change “Note” to “For the purpose of this example, we say”. If one wants to retain the more standard convention that the empty set spans  $\{0\}$ , then we need to enlarge  $L_n(q)$  by adding  $\emptyset$  below  $\{0\}$ .
- p. 128, line 14. The type of a set partition  $\pi \in \Pi_n$  has not been defined, though the definition should be clear in analogy to the type of a permutation. Namely, define  $\text{type}(\pi) = (a_1, \dots, a_n)$  if  $\pi$  has  $a_i$  blocks of size  $i$ .

- p. 128, line 16. Change  $=$  to  $\cong$ .
- p. 131, line 6 of text. Change “rank  $i$ ” to “of rank  $i$ ”.
- p. 131, line 5–. Change  $\sigma : P \rightarrow [n]$  to  $\sigma : P \rightarrow \mathbf{n}$ .
- p. 133, line 13–. Change  $a_0 = \hat{0}, a_{s+1} = \hat{1}$  to  $a_0 = 0, a_{s+1} = n$ .
- p. 136, line 6. Change  $(-1)^n \Delta Z(P, m - 1)$  to  $(-1)^{n-1} \Delta Z(P, m - 1)$ .
- p. 137, line 7. Change  $Q$  to  $P - Q$ .
- p. 142, line 14–. It should have been stated that  $\text{card}(x, y)$  is short for  $\text{card}([x, y])$ .
- p. 153, Exercise 1a, line 1. Change “operation” to “relation”.
- p. 155, line 4. Insert “irreducible” before “*connected*”. (An irreducible poset is one that cannot be written in a nontrivial way as a direct product.)
- p. 156, Exercise 15(d). This exercise was solved by J. D. Farley and R. Klippenstine, Posets with the same number of order ideals of each cardinality, II, preprint dated November 30, 2004.
- p. 157, Exercise 22(e). It was shown by J. Farley, *J. Combinatorial Theory (A)* **90** (2000), 123–147, that the only nondecreasing cover functions (with  $f(0) \geq 1$ ) are  $f(n) = k$  and  $f(n) = n + k$  for  $k \geq 1$ . This confirms a conjecture of R. Stanley, *Fibonacci Quart.* **13** (1975), 215–232 (page 226).
- p. 160, line 1. Change second  $P$  to  $\bar{P}$ .
- p. 160, Exercise 32, line 4. Insert  $\mu(x_k, \hat{1})$  after  $\mu(x_{k-1}, x_k)$ .
- p. 164, line 1. Change  $a_i$  to  $y$  (twice).
- p. 166, line 1–. Change  $X_1$  to  $H_1$  and  $X_\nu$  to  $H_\nu$ .
- p. 167, Exercise 59a. Change the last sentence to: Show that if  $Z(P, m + 1) = \sum_{i \geq 1} a_i \binom{m-1}{i}$ , then  $Z(Q_0, m + 1) = 1 + \sum_{i \geq 1} a_i m^i$ .
- p. 174, Exercise 81a, lines 3–4. Change “define  $f, g$ ” to “define  $g, h$ ”.
- p. 174, Exercise 81c, line 5. Change this line to

$$1 + t \sum_{n \geq 1} G_n(q, t) x^n / (\mathbf{n})! = \left[ 1 - t \sum_{n \geq 1} (1 - t)^{n-1} x^n / (\mathbf{n})! \right]^{-1}.$$

- p. 174, Exercise 81c, line 2–. Change  $(1 - t)/(e^{x(t-1)} - 1)$  to  $(1 - t)/(e^{x(t-1)} - t)$ .

- p. 178, solution to Exercise 19a, lines 5–7. Interchange  $f_k$  and  $g_k$ .
- p. 184, solution to Exercise 32, line 3. Insert  $\mu(x_k, \hat{1})$  after  $\mu(x_{k-1}, x_k)$ .
- p. 184, solution to Exercise 37b, line 1. Change “ $f(x, s) = x$ ” to “ $f(x, s) = \phi(x)$  (so  $F(x, s) = x$ )”.
- p. 187, line 2. Remove  $\prod_{i=1}^k$ .
- p. 187, line 3–. Change  $x^{n-\dim W}$  to  $x^{n-\dim W'}$ .
- p. 188, Exercise 46, second solution, lines 1–2. Change  $a \neq \{1\}$  to  $\emptyset \neq a$ .
- p. 191, Exercise 50, line 4. Under the third summation symbol, change  $x \in P_j$  to  $y \in P_j$ .
- p. 191, Exercise 51, line 5. Change “voltage graphs” to “gain graphs”.
- p. 191, Exercise 51, line 7. A general reference for enumerative results on gain graphs is T. Zaslavsky, *J. Combin. Theory Ser. B* **64** (1995), 17–88.
- page 192, line 2–. This formula should be:

$$Z(P \oplus Q, m) = Z(P, m) + Z(Q, m) + \sum_{j=2}^{m-1} Z(P, j)Z(Q, m+1-j), \quad m \geq 2.$$

- p. 193, line 1. Change “chains” to “multichains”.
- p. 195, solution to Exercise 61b. Interchange  $\mathbf{p}$  and  $p\mathbf{1}$  throughout.
- p. 206, first line of proof. Change  $R(x)$  to  $F(x)$ .
- p. 216, line 5. Change 1.3.3 to 1.3.
- p. 224, line 3. Change “rank  $d = \dim \mathcal{C}$ ” to “rank  $d + 1$  where  $d = \dim \mathcal{C}$ ”.
- p. 227, line 8. Change “ $a_i = \lceil b_i \rceil$ , the least integer  $\geq b_i$ ” to “ $a_i = \lceil b_i - 1 \rceil$ , where  $\lceil b_i \rceil$  denotes the least integer  $\geq b_i$ ”.
- p. 230, line 2 of Proof. Change  $d - \dim \sigma$  to  $d - \dim \sigma + 1$ .
- p. 231, Lemma 4.6.17(i). As stated, the result is false. For instance, let  $E = \mathbb{N}$  and  $a_1 = -1$ . Then  $G(\lambda) = 1$  but  $E(\lambda^{-1}) = \sum_{n \geq 0} \lambda^{-n}$ . We need to assume also that  $g(r) > 0$  for at least one  $r > 0$ . We claim that then  $g(s) = 0$  for all  $s < 0$ , and hence (i) follows. Let  $\alpha \in E$  satisfy  $L(\alpha) = r > 0$ , and suppose that there exists  $\beta \in E$  with  $L(\beta) = s < 0$ . Then for all  $t \in \mathbb{N}$  the vectors  $ts\alpha + tr\beta$  are distinct elements of  $E$ , contradicting  $g(0) < \infty$ .

- p. 236, line 7. Change “union” to “intersection”.
- p. 237, line 8. Change  $\text{den}(\gamma, t)$  to  $\text{den}(\gamma/t)$ .
- p. 244, Figure 4-13. Add an edge from 13 to 31.
- p. 246, lines 5–6. Change 7 to 6 (twice).
- p. 249, line 15–. Add at end of line: where  $b(n) = \sum_{v \in \mathcal{B}_n} w(v)$ .
- p. 253, line 4–. Change  $D_4$  to  $D_3$ .
- p. 256, line 2. Change  $(j - 1, j)$  to  $(j, j - 1)$ .
- p. 257, line 19–. Change  $f_s(n_s)$  to  $f_k(n_k)$ .
- p. 257, line 18–. Change  $f_s(n_{s+1})$  to  $f_k(n_{k+1})$ .
- p. 260, Figure 4-42. Change the label on the edge from 00 to 01 from  $F_1 * F_2$  to  $F_1 * F_3$ .
- p. 262, line 2–. The result of Hickerson has now appeared in *J. Integer Sequences* (electronic) **2** (1999), Article 99.1.8,

<http://www.research.att.com/~njas/sequences/JIS/HICK2/chcp.html>.

- p. 266, Exercise 12. Change  $\Phi\alpha = \mathbf{0}$  to  $\Phi\alpha = \beta$ , where  $\beta$  is a vector of linear polynomials  $an + b$ . Moreover, the final sentence should be “Show that  $\mathbb{N}^m$  can partitioned into finitely many regions such that on each region the number of solutions is a quasipolynomial in  $n$  for  $n$  sufficiently large.” It is possible that this result is already known.
- p. 271, Exercise 27(a), line 2. Change this line to

$$x_1 + x_2 + \cdots + x_r \leq 1, \quad y_1 + y_2 + \cdots + y_s \leq 1, \quad x_i \geq 0, \quad y_i \geq 0.$$

- p. 286, Exercise 23. Peter McNamara has pointed out a gap in the proof. Namely, from the fact that  $P - M$  and  $P - M'$  are disjoint unions of chains, it need not follow (when  $P$  has maximal chains of length one) that  $P$  is a disjoint union of chains, together with the stated relations  $x < y$ . To fix the proof, note that  $m$  is the largest power of  $x_0$  that can appear in a monomial in  $\bar{G}(P, \mathbf{x})$ . Hence  $m$  is the largest power of *any*  $x_i$  that can appear in a monomial in  $\bar{G}(P, \mathbf{x})$ . Let  $A$  be an antichain of  $P$ . We can easily find a strict  $P$ -partition that is constant on  $A$ , so  $\#A \leq m$ . Hence the largest antichain of  $P$  has size  $m$ . By Dilworth’s theorem,  $P$  is a union of  $m$  chains. Each such chain intersects  $M$  and  $M'$ . It is now easy to see that if  $P - M$  and  $P - M'$  are disjoint unions of chains, then indeed  $P$  is a disjoint union of chains together with the stated relations  $x < y$ , and the proof proceeds as before.

- p. 293, line 9–. In the definition of a connected graph, it should also be specified that the empty graph is *not* connected.
- p. 295, Figure A-2. Interchange the labels 3 and 5 on the third tree.
- p. 308, Problem 7, line 3. Change  $[n]$  to  $[n - 1]$ .
- p. 310, Problem 25, line 2. Change “occurences” to “occurrences” (twice).
- p. 310, Problem 25, line 3. Change “ $t$  occurences of each  $a_{ij}$ ” to “ $2t$  occurrences of each  $a_{ij}$ ”.
- p. 314, Problem 6(c). A solution was found by Ethan Fenn (private communication, November, 2002). The rating should be changed to [3–] and the problem restated as follows.

Let  $k \geq 3$ , and let  $P_k$  denote the poset of all subsets of  $\mathbb{P}$  whose elements have sum divisible by  $k$ . Given  $T \leq S$  in  $P_k$ , let

$$i_j = \#\{n \in T - S : n \equiv j \pmod{k}\}.$$

Clearly  $\mu(S, T)$  depends only on the  $k$ -tuple  $(i_0, i_1, \dots, i_{k-1})$ , so write  $\mu(i_0, \dots, i_{k-1})$  for  $\mu(S, T)$ . Show that

$$\begin{aligned} & \sum_{i_0, \dots, i_{k-1} \geq 0} \mu(i_0, \dots, i_{k-1}) \frac{x_0^{i_0} \cdots x_{k-1}^{i_{k-1}}}{i_0! \cdots i_{k-1}!} \\ &= k \left[ \sum_{j=0}^{k-1} \exp(x_0 + \zeta^j x_1 + \zeta^{2j} x_2 + \cdots + \zeta^{(k-1)j} x_{k-1}) \right]^{-1}, \end{aligned}$$

where  $\zeta$  is a primitive  $k$ th root of unity.

- p. 314, Problem 10, line 1. Delete the difficulty rating [2+] at the beginning of the line.
- p. 314, Problem 13, lines 2–3. Delete the difficulty rating [2+] at the beginning of these lines.
- p. 315, lines 3– and 2–. Delete the difficulty rating [2+] at the beginning of these lines.
- p. 315, line 2–. Delete “(ii)”.
- p. 318, Problem 13. This should be rated [2+].
- p. 319, line 9–. This erratum is unnecessary and can be deleted.
- p. 320, line 2 (paperback edition only). Change  $(2^{a_1} - 1) \cdots (2^{a_1} - 1)$  to  $(2^{a_1-1} - 1) \cdots (2^{a_k-1} - 1)$ .

- p. 320, lines 16–17. Delete this item.
- p. 321, item 3, line 2. Change  $\mu(kn)$  to  $\mu_S(kn)$ .
- p. 321, item 6. The computation of  $f(14) = 1338193159771$  is given by J. Heitzig and J. Reinhold, *Order* **17** (2000), 333–341. They also compute the number of *labelled*  $n$ -element posets for  $n \leq 16$ . According to Vledeta Jovović, as reported on the *Encyclopedia of Integer Sequences*, A000112, <http://www.research.att.com/~njas/sequences>, we have  $f(15) = 68275077901156$  and  $f(16) = 4483130665195087$ .
- p. 321, line 7. Delete this entry (for p. 149, line 10).
- p. 322, lines 8– to 5–. The stated result is false. The hypothesis on  $L$  should be the following:  $L$  is an  $n$ -element lattice such that for all  $\hat{0} < x \leq y$  in  $L$ , there exists  $z \neq y$  such that  $z \vee x = y$ .
- p. 322, line 4–. Change  $\ell^{k-1}$  to  $k^{\ell-2}$ .