ERRATA
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• page 5, line 9–. Insert “a” after “is”.
• page 23, line 6. Insert “a” after “is”.
• page 24, line 2. Change $c_i$ to $c_1$.
• page 34, line 15. It should be stated that for this second way, $d + 1$ cannot be inserted at the end even though $d \notin D(w)$.
• page 38, line 18. Change “does” to “do”.
• page 40, line 15. The intent of the parenthetical comment was to write the summation as

$$\sum_{g(4) \geq 0} \sum_{g(6) \geq g(3)} \cdots \sum_{g(5) \geq 0} q^{g(4)+\cdots+g(5)}.$$ 

We are actually first summing on $g(5) \geq 0$, then $g(1) \geq g(5)$, etc.
• page 40, line 4–. Insert $\sum_{m \geq 0}$ before $\sum_{w \in Sd}$.
• page 43, line 4. Change “relection” to “reflection”.
• page 43, line 14. Change $i \geq 0$ to $i \geq 1$.
• page 45, line 1. Change $w$ to $T(w)$.
• page 47, line 11. Change $n \geq 2$ to $n \geq 1$.
• page 49, lines 20–22. The two sentence beginning “Figure 1.10 . . .” and “Let $f(n)$ denote . . .” should be interchanged, since $f(n)$ is used in the first of these sentences but defined in the second.
• page 50 , line 4. Change “tree” to “trees”.
• page 53, line 1–. Change $n$ to $n - 1$.
• page 54, line 8. Change “number” to “numbers”.


- page 62, equation (1.78). Change $j \in S_i$ to $j \in S_i$.
- page 62, line 13–. Change $S_j$ to $S_i$.
- page 65, line 7. Change proposition to Proposition.
- page 66, line 13. Change $F(0,q)$ to $F(x,0)$.
- page 68, line 17–. Change “$k$ of the $\gamma_i$’s” to “$k \gamma_i$’s”.
- page 71, line 2–. Change “then then” to “then the”.
- page 76, line 2. Change “$x$ to $-x$” to “$t$ to $-x$”.
- page 77, line 5–. Change “$f(0), f(0)$” to “$f(0), f(1)$”.
- page 85, Theorem 1.10.4, line 3. Change $GL(n,q)$ to $GL(m,q)$.
- page 85, Theorem 1.10.4, line 4. Change $|\lambda(f)| \cdot \deg(f)$ to $m = |\lambda(f)| \cdot \deg(f)$.
- page 88, line 13–. Change $v_{ij}$ to $v_{ij}$.
- page 98, line 7. Change “in the monograph” to “is the monograph”.
- page 109, Exercise 35(b,c,d). For the history of these results, see MathOverflow 63561.
- page 112, Exercise 54(a), line 1. Change $A(2n+1, n+1)$ to $A(2n+1, n)$.
- page 112, Exercise 55, line 2. Delete “, respectively” since $\text{maj}(w^r)$ is expressed in terms of both $\text{des}(w)$ and $\text{maj}(w)$.
- page 113, Exercise 1.58(a), line 2. Change “and” to “can”.
- page 115, Exercise 70(b), line 3. Change $(1,0)$ to $(0,1)$, and change “i.e,” to “i.e.,”.
- page 120, Exercise 94, second bullet. Change “Then” to “The”.
- page 121, Exercise 96(c), line 3. Change 4.4.1.1 to 4.1.1.
- page 123, Exercise 116(b), line 2. Change distribuiton to distribution.
• page 126, line 2. Change $1 \leq i < j \leq n$ to “$1 \leq i \leq j \leq n$, excluding $(i, j) = (1, n)$.”

• page 126, Exercise 128(d), line 7. Change $\frac{2}{1+x}$ to $\frac{2x}{1+x}$.

• page 126, Exercise 129(a), line 3. Add $\sum_{k \geq 0}$ after $\sum_{n \geq 1}$.

• page 132, Exercise 160(c), lines 2,3. Change $\xi$ to $\zeta$.

• page 136, line 1–. Change $q^{n-1}$ to $q^{k-1}$.

• page 137, Exercise 184(a), line 5. Change “polynomials $f(x) = \beta$ for $0 \neq \beta \in \mathbb{F}_q$” to “polynomial $f(x) = 1$”.

• page 139, Exercise 193, line 2. Change $\gamma_n$ to $\gamma(n)$.

• page 140, Exercise 199, line 4. The condition (only relevant in characteristic 2) should be that $A = -A^t$ and $A$ has 0’s on the main diagonal. Some people will take this to be the definition of “skew-symmetric,” while others will call these matrices “alternating.”

• page 140, Exercise 198(b), last displayed equation. Change $(q^{2m-1})$ to $(q^{2m-1} - 1)$ (twice).

• page 148, Exercise 56, line 3. Change $=$ to $\subseteq$.

• page 150, Exercise 22(b), line 6. Change $x^{2i+1}$ to $x^{2i-1}$.

• page 156, line 2. Change Combinatorica to Combinatoria.

• page 163, Exercise 80. For the convoluted history of this result, see R. Gilbert, A Fine rediscovery, Amer. Math. Monthly 122 (2015), 322–331. In particular, the result is actually first due to N. J. Fine in 1959.

• page 166, Exercise 91(d), line 4. Replace this line with

$$1 + \sum_{n \geq 1} (-1)^n (x^n - x^{-n-1})q^{n(n-1)/2}.$$  

• page 166, Exercise 92. This identity is actually due to I. Gessel and D. Stanton, Trans. Amer. Math. Soc. 277 (1983), 173–201 (equations (7.13) and (7.15)).
• page 169, Exercise 109(a), line 4. Change “is bijection” to “is a bijection”.

• page 171, line 4. The index entries for these four names is missing.

• page 173, Exercise 129. Change the parts a and b to b and c.

• page 173, Exercise 134, line 2. Change the period after Petersen to a comma.

• page 181, Exercise 164, line 1. Change $x = 0$ to $y = 0$.

• page 186, Exercise 164. Change the parts a and b to b and c.

• page 186, Exercise 185, line 6. Insert $\prod_{d \geq 1}$ after $=.$

• page 190, Exercise 1.190(b), line 3. Replace this line with

$$\omega^*(n, q) = q^n - q^m - q^{m-1} - q^{m-2} - \cdots - q^{\lfloor n/3 \rfloor} + O(q^{\lfloor n/3 \rfloor-1}).$$

• page 195, line 9–. Change “example” to “examples”.

• page 196, equation (2.5). Change $Y \supseteq T$ to $Y \supseteq \emptyset$ or to just $Y$.

• page 198, line 2. This line should read:

function $f_\geq$ satisfies $f_\geq(T) = f_\geq(T')$ whenever $\#T = \#T'$. Thus also $f_\geq(T)$

• page 199, line 11–. Change “i set” to “i-set”.

• page 199, line 3–. Change $S - T$ to $S_n - T$.

• page 200, line 8. Change $j \geq i$ to $j \leq i$.

• page 203, line 6. Insert ) before }.

• page 203, line 13. Change “nonatttacking” to “nonattacking”.

• page 203, line 4–. Change “is” to “in”.

• page 205, line 2–. Change $n - k - 1$ to $n - k + 1$. 
• page 206, line 9. Change “suggest” to “suggests”.
• page 207, line 4–. Change $x - s_1$ to $x + s_1$.
• page 208, line 10–. Change “a3’s” to “a3 3’s”.
• page 210, line 9. Insert “one for 121,” after “one for 112,”.
• page 210, line 9–. Insert = between $d(n)$ and $\#D_n$.
• page 212, line 9–. Change $T \subseteq S$ to $Y \subseteq S$.
• page 216, line 5–. Insert ) after $x_{\gamma_i}$.
• page 217, l. 14–. Change v to $(x, y)$.
• page 217, l. 12–. Change v to $(x, y)$.
• page 222, Exercise 10(b), lines 1 and 2. Change $E(n)/n!$ to $E(n)/n^n$ (twice).
• page 227, line 3–. On the left-hand side of the formula, remove one $\sum_{i \geq 0} a_n$ and change $y^i$ to $t^i$.
• page 229, Exercise 29(a), line 4. Change “partitions” to “compositions”.
• page 235, Exercise 10(b), line 9. Change $E(n)/n!$ to $E(n)/n^n$.
• page 237, Exercise 19. Change $\binom{2n-k}{k}$ to $\frac{2n}{2n-k} \binom{2n-k}{k}$.
• page 237, line 2–. Change second $a_{n-k+1}$ to $a_{n-k+2}$.
• page 248, line 5–. Change “an” to “a”.
• page 249, Proposition 3.3.2, line 1. Change “conditions” to “conditions”.
• page 252, line 3. Insert “simple” before “matroids”.
• page 253, line 9–. In the definition of finitary distributive lattice, add the condition that $L$ has finitely many elements of each rank. (Thus for instance the distributive lattice of all finite subsets of an infinite set is locally finite with $\hat{0}$ but not finitary.)
• page 277, line 12–. Change \( \dim(W \cup W') \) to \( \dim(W + W') \).

• page 283, lines 8– to 6–. Delete the sentence “Let \( \Lambda_t = \{ s \in L(A) : s \leq t \} \), the principal order ideal generated by \( t \).”

• page 284, line 3–. Delete “real” at the end of the line.

• page 284, line 2–. Insert “over \( K \)” after “arrangements”.

• page 285, line 9. Delete “\#B_1 = \#B - 1 and”.

• page 285, line 11. Under the second \( \Sigma \), change \( B_1 \in A'' \) to \( B_1 \subseteq A'' \)

• page 291, line 10–. Change “sketch that” to “reference for”.

• page 303, Theorem 3.15.8. To be completely accurate, one should assume that \( P \neq \emptyset \).

• page 308, line 9. Change “an” to “and”.

• page 313, Figure 3.33. The poset \( P_3 \) is drawn incorrectly.

• page 315, line 7. Change \( a_s - a_{s+1} \) to \( a_s - a_{s-1} \) (twice after the brace).

• page 317, after (3.80). The phrase “(since intervals of Eulerian posets are Eulerian)” is unnecessary since the formulas under consideration hold for any graded poset with \( \hat{0} \) and \( \hat{1} \).

• page 319, line 12–. Change \( Q_1 \ast Q_2 \ast \cdots \ast Q_r \) to \( R_1 \ast R_2 \ast \cdots \ast R_r \) (since \( Q_i \) already has been given another meaning).

• page 319, line 11–. Change the first \( Q_i \) to \( R_i \).

• page 332, line 1–. Change \( \gamma_{j-1} \) to \( \gamma_{j-1}^2 \).

• page 332, bottom of page. Add the following paragraph:

The proof that \( (\gamma_j^*)^2 = 1 \) is completely analogous. It also follows from the fact that \( \gamma_j^2 = 1 \), since we can assume without loss of generality that \( j = p - 1 \) and then apply the automorphism of \( G \) that sends each \( \tau_k \) to \( \tau_{p-k} \).

• page 334, line 4. Change \( z\delta \) to \( z\delta_{p-1} \).
• page 336, line 11. Change \( \varphi: \hat{KP} \to \hat{KP} \) to \( \varphi: KP \to \hat{KP} \).

• page 337, lines 14– to 11–. The sentence “For the algebraically minded ... formal power series.” is not correct. We first need to consider non-commutative *polynomials* (not power series) and then pass to suitable completions.

• page 342, line 6. Change \((1 - q)^2\) to \((1 - q)\) (in the denominator).

• page 345, line 4. Change \(p_{s-1} + p_s\) to \(p_{j-1} + p_j\).

• page 345, line 13. Change \(A\) to \(A\) (boldface).

• p. 356, Exercise 15(g), line 3. Change 9655 to 11586.


• page 360, Exercise 38. This exercise is incorrect for \(1 < k < m\), where \(m\) is the maximum size of an antichain of \(P\). A counterexample is given by \(k = 2\) and \(L = J(P)\), where \(P = 1 + (1 \oplus (1 + 1))\). It is true that \(\#P_k = \#R_k\), even for modular lattices. See Exercise 3.101(d).

• page 360, Exercise 41(c,d). It is assumed that \(L\) is distributive.

• page 364, Exercise 55(b). The rating should be changed to [3–]. An exceptionally elegant proof was given by G. Stachowiak, *Order* 5 (1988), 257–259. Another elegant proof was given by B. Iriarte, arXiv:1405.4880. Iriarte proves the more general result that if \(G\) is the comparability graph of a poset \(P\), then the number of linear extensions of \((G, o)\) is maximized when \(o\) respects the order \(P\).

• page 374, Exercise 95. The stated identity is actually valid for the dual Möbius algebra, where one uses the join operation rather than the meet. The correct identity using the text definition of Möbius algebra is

\[
\sum_{t \in L} \mu(t, \hat{1})t = \left( \sum_{u \geq z} \mu(u, \hat{1})u \right) \cdot \left( \sum_{v \lor z = \hat{1}} \mu(v, \hat{1})v \right).
\]

• page 375, Exercise 98, line 3. Change \(\leq\) to \(\geq\).
- page 378, Exercise 114(a). The answer should be
  \[(x - 1)(x - 2) \cdots (x - n).\]

- page 379, Exercise 115(c), line 1. Change “0,1” to 0. (There are \(\binom{n}{2}\) hyperplanes.)

- page 379, Exercise 116. The subscripts should not be in boldface.

- page 387, Exercise 3.139. The right-hand side of the displayed formula should be \(2x(1 + x)^{n-2}/(1 - x)^n\).

- page 390, Exercise 158(a), line 4. Change a. at beginning of line to i.

- page 390, Exercise 158(a), line 5. Change b. at beginning of line to ii.

- page 390, Exercise 158(c). Delete one of the periods at the end of the line.

- page 390, Exercise 158(d), line 2. Change “thats” to “that”.

- page 403, Exercise 189. NOTE. The result is also true for \(d\) odd, but the proof is quite a bit easier.

- page 404, Exercise 198. It should be mentioned that butterfly posets are also called “ladder posets,” since there is an index entry for “poset, ladder.”

- page 405, line 8-. The notation \(f_{00}\) is not defined until the next sentence.

- page 407, Exercise 205(b), line 2. Change 44605 to 44606 (private communication from Patrick Byrnes, 21 February 2012). Byrnes originally assumed that a vertex \(v\) could be covered by at most one singleton (element covering only \(v\)), but there is exactly one example up to rank 9 where this property need not hold. Byrnes also computes that there are 29,199,636 1-differential posets up to rank 10.

- page 408, Exercise 211, line 3. Change the numerator \(r^2 + (r + 1)q - q^2\) to \(r^2 + r(r + 1)q - rq^2\).

• page 415, Exercise 36(b). This example is incorrect. For instance, \{1, 2, 3\} and \{1, 2, 4, 5\} have no meet.

• page 431, line 2–. Change 23 to 33.

• page 449, Exercise 163(a), line 3. Change \(2e_P(p - 1) - (p - 1)e(p)\) to \((2e_P(p - 1) - (p - 1)e(P))/p!\).

• page 458, Exercise 185. Part f. should be k., and g. should be l.

• page 459, line 5–. Should be \(\beta_P(4, 5, 6) = -1\).

• page 465, line 7–. Change “degree j” to “degree j – 1”.

• page 468, Proposition 4.2.2, line 5. Change the period after \(E_f\) to a comma.

• page 469, line 6–. Change \(\sum_{n \geq 0} f(n)\) to \(\sum_{n \geq 0} f(n)x^n\).

• page 511, line 18–. Change \((v, b_i)\) to \((v, i), 1 \leq i \leq k\).

• page 522, Figure 4.29. The vertex labels should be 0 and 1, not 00 and 10.

• page 528, Exercise 4.2, line 3. Change “over” to “in”.

• page 530, Exercise 4.12. Change 0.0010203050813213455\ldots to 0.0001010203050813213455\ldots, and change 1, 2, 3 to 1, 1, 2, 3.

• page 530, Exercise 4.14, line 2. Insert “relatively prime” after “choose a, b”.

• page 531, Exercise 22, line 1. Change \(\alpha \in \mathbb{C}\) to \(0 \neq \alpha \in \mathbb{C}\).

• page 532, Exercise 25(e), line 3. Change \(N_1(m)\) to \(N_1(2^m - 1)\).

• page 532, Exercise 25(e), line 5. Change \(N_1(m)\) to \(N_1(q^m - 1)\) and \(N_{-1}(m)\) to \(N_{-1}(q^m - 1)\).

• page 532, Exercise 25(e), line 8. Change \(N_1(m)\) to \(N(2^m - 1)\).

• page 532, Exercise 25(e), line 10. Change \(N_1(m)\) to \(N_1(2^m - 1)\).

• page 533, line 2. Change \(N_1(m)\) to \(N_1(2^m - 1)\).
• page 533, line 5. Change $N_1(m)$ to $N_1(2^m - 1)$.
• page 533, line 7. Change $N_1(m)$ to $N_1(3^m - 1)$.
• page 533, line 8. Change $N_2(m)$ to $N_2(3^m - 1)$.
• page 533, line 10. Change $N_1(m)$ to $N_1(3^m - 1)$.
• page 533, line 11. Change $N_2(m)$ to $N_2(3^m - 1)$.
• page 539, Exercise 47, line 1–. Change 1.1.8.6 to 1.8.6.
• page 542, line 4. Change $n \times n$ to $(n + 1) \times (n + 1)$.
• page 542, line 10. Delete ) after $u_6$.
• page 546, Exercise 4.75(d). Change the difficulty rating to [3–]. A combinatorial proof (though not a simple bijection) was found by Jacob Scott, private communication dated September 2, 2012.
• page 548, Exercise 2(a), line 6. The assertion “Clearly, we can write $f(x) = P(x)/Q(x)$ for some relatively prime integer polynomials $P$ and $Q$.” is not so clear. One must show that if $F(x) \in \mathbb{Q}[x]$ and $F(x) = R(x)/S(x)$ where $R, S \in \mathbb{C}[x]$, then one can write $F(x) = P(x)/Q(x)$ where $P, Q \in \mathbb{Q}[x]$. This statement does have a fairly simple proof, which we leave as an exercise.
• page 549, line 1–. Change $x^{m^2}$ to $x^{n^2}$.
• page 552, Exercise 14. The smallest known pair $(a, b)$ seems to be
\[
\begin{align*}
a & = 106276436867 = 31 \cdot 3128272157 \\
b & = 35256392432 = 2^4 \cdot 2203524527
\end{align*}
\]
due to M. Vsemirnov, *J. Integer Seq.* 7 (2004), article 04.3.7.
• page 557, line 4–. Change $x^k$ to $x^l$.
• page 558, line 1. Delete comma after *Wochenschrift*.
• page 558, Exercise 43. The sequence $t(3), t(4), \ldots$ is known as *Alcuin’s sequence*, after Alcuin of York (730s or 740s – 19 May 804). For a survey see D. J. Bindner and M. Erickson, *Amer. Math. Monthly* 119 (2012), 115–121.
• page 560, Exercise 46(b), line 1. Delete the first “many”.

• page 560, Exercise 46(b), line 2. Change \( f(n+1), f(n+2), \ldots, f(n+d) \) to “\((f(n+1), f(n+2), \ldots, f(n+d))\)” for fixed \( d \).

• page 564, Exercise 58(f), line 1. Change “order polynomials” to “Ehrhart polynomials”.

• page 567, Exercise 75(c), line 1. Delete “directed”.

• page 569, Exercise 80, displayed equation, line 1. Change \( v'_iy \) to \( v_iy \).

• page 570, Exercise 82(e,f). For the paper of J. Schneider, see Electronic J. Combinatorics 21 (2014), P1.43; arXiv:1206.6174.

• page 571, line 3. Change “chose” to “chosen”.

• page 578. The following item is missing: First edition—Supplementary Exercise 1.7 = Second edition—Exercise 1.114(b).