Here I will maintain supplementary material for *Enumerative Combinatorics*, volume 2 (paperback edition of 2001). This will include errata, updated references, and new material. I will be continually updating this supplement.

**Note.** References to math.CO refer to the combinatorics section of the xxx Mathematics Archive at xxx.lanl.gov/archive/math. A front end site for math.CO is front.math.ucdavis.edu/math.CO.

- p. 6, line 10. Change situtations to situations.
- p. 11, line 3. Change $E_c(n)$ to $E_c(x)$.
- p. 18, line 3. Change $(n)_2$ to $n(n-2)$.
- p. 20, line 9. Change $Z(S_n)$ to $\tilde{Z}(S_n)$.
- p. 24, line 4 (after figure). Change $\lim_{n\to\infty}$ to $\lim_{k\to\infty}$.
- p. 25, line 5. Change $\subseteq$ to $\in$.
- p. 33, line 5–. Change $\text{ord}(\tau_k)$ to $\text{ord}(\tau_j)$.
- p. 34, Lemma 5.3.9. Delete the first sentence, viz., “Let $w \in A^*$.”
- p. 35, line 10. Change $w \in B^*$ to $w \in B_r^*$.
- p. 35, line 8–. Change $\mathcal{A}$ to $\mathcal{A}$.
- p. 36, lines 15–16. Change “beginning with a 1” to “ending with a $-1$”.
- p. 36, line 1–. Insert $\ldots$ before $=$. (The left-hand side is an infinite sum.)
• p. 51, line 9–. Change \( Q_i = \Pi_i^{(2)} \) to “when \( Q_i \) is given by Example 5.5.2(d) for \( r = 2 \).

• p. 59, line 9. Change “Since the rows” to “Since the columns”.

• p. 59, line 13. Change “Because the columns” to “Because the rows”.

• p. 62. Example 5.6.12, line 5. Change “modulo \( n \)” to “modulo \( 2^n \)”.

• p. 63, line 12. Change “sequence” to “sequences”.

• p. 65, line 8. Change “Theorem” to “Lemma”.

• p. 87, equation (5.111). We need to add the further condition that \( p_n(0) = \delta_0n \). Otherwise, for instance, the polynomials \( p_n(x) = (1 + x)^n \) satisfy (iv) with \( Q = \frac{d}{dx} \) but fail to satisfy (i)–(iii).

• p. 97, Exercise 5.51. It is not true that (ii) implies (i), e.g., when \( C(x) = c \). One needs to add the hypothesis that \( [x]C(x) \neq 0 \), so that \( (C(x) - c)^{(-1)} \) exists. Substituting \( xC(B(x)) \) for \( x \) in (ii) yields

\[
xC(B(x))/C(A(xC(B(x)))) = x,
\]

so \( C(B(x)) = C(A(xC(B(x)))) \). Substituting \( B(x)^{(-1)} \) for \( x \) yields \( C(x) = C(A(B(x)^{(-1)}C(x))) \). Subtract \( c \) from both sides and apply \( (C - c)^{(-1)} \) to get \( x = A(B(x)^{(-1)}C(x)) \). Applying \( A^{(-1)} \) to both sides gives (i). This argument is due to Daniel Giambio and Amit Khetan and (independently) to Yumi Odama.

• p. 101, line 3. Change \( J_0[(2 - t)/\sqrt{t - 1}] \) to \( J_0 \left( \sqrt{-t} (2 - t)/(1 - t) \right) \).

• p. 102, Exercise 5.71. It would be better not to specify the degree \( d \) of \( G \), since (as stated in the solution) \( d = \lambda_1 \).

• p. 103, Exercise 5.74(d). Replace the first two sentences with: Show that all vertices have the same outdegree \( d \). (By (c), all vertices then also have indegree \( d \).)

• p. 108, Exercise 5.7(a), line 7. Change $b_{2n}$ to $b_{2n-k}$.

• p. 137, Exercise 5.45, line 1. Change $kxy^k$ to $(k+1)xy^k$.

• p. 137, Exercise 5.45, line 4. Change this equation to

$$y = x + 2xy + 3xy^2 + \cdots = \frac{x}{(1-y)^2}.$$  


• p. 147, Third Solution. The first two lines should be: Equation (5.530) can be rewritten (after substituting $n+k$ for $n$)

$$\frac{(n+k)[x^n]}{k} \left( \frac{F^{(-1)}(x)}{x} \right)^k = [x^n] \left( \frac{x}{F(x)} \right)^{n+k}.$$  

(5.140)

• p. 162, lines 13– to 12–. Change “Thus any algebraic power series, as defined in Definition 6.1.1” to “Thus any algebraic function, i.e, any solution $\eta$ to (6.2)”.

• p. 175, line 1. Change $\{9,11\}$ to $\{9,14\}$.

• p. 175, line 2. Change $x^{11}$ to $x^{14}$.

• p. 175, line 4. Change $v^{11}$ to $v^{14}$.

• p. 175, line 11. Change Theorem 5.4.1 to Theorem 5.4.2.

• p. 175, line 2–. Change $k \in K$ to $k \in \mathbb{Z}$.

• p. 176, line 16. Change intesect to intersect.

• p. 176, line 4–. Change $(n+2)$-gon to $(n+1)$-gon.

• p. 192, line 9–. Change $u(0) = 0$ to $v(0) = 0$.  


• p. 192, lines 8– to 7–. The example $v = \log(1 + x^2) - 1$ is confusing since $v(0) \neq 0$. Nevertheless the series $u(v(x)) = \sqrt{\log(1 + x^2)}$ is well-defined formally since we can write

$$\sqrt{\log(1 + x^2)} = x \sqrt{\frac{\log(1 + x^2)}{x^2}}.$$ 

It would have been more accurate to define

$$v(x) = \frac{\log(1 + x^2)}{x^2} - 1.$$ 

The same remarks apply to Exercise 6.59.


• p. 212, lines 16–17. I have forgotten the source for the statement that Netto was the first to use the term “Catalan number.” Can anyone provide a reference?

• p. 219, Exercise 6.16. A combinatorial proof was first given by R. Sulanke, *Electronic J. Combinatorics* 7(1), R40, 2000. A sharper result was subsequently proved combinatorially by D. Callan, A uniformly distributed parameter on a class of lattice paths, math.CO/0310461.

• p. 221, Exercise 6.19(j). This problem appeared as Problem A5 on the 2003 William Lowell Putnam Mathematical Competition. Many participants found the following bijection with 6.19(i) (Dyck paths from $(0, 0)$ to $(2n, 0)$): Let $D$ be a Dyck path from $(0, 0)$ to $(2n, 0)$. If $D$ has no maximal sequence of $(1, -1)$ steps of even length ending on the $x$-axis, then just prepend the steps $(1, 1)$ and $(1, -1)$ to the beginning of $D$. Otherwise let $R$ be the rightmost maximal sequence of $(1, -1)$ steps of even length ending on the $x$-axis. Insert an extra $(1, 1)$ step at the beginning of $D$ and a $(1, -1)$ step after $R$. This gives the desired bijection.
• p. 224, item ii, line 5. Change $S(w) = w$ to $S(w) = 12 \cdots n$.

• p. 228, item iii, line 3. To be precise, the displayed sequences should have the initial and final 1’s deleted.

• p. 230, Exercise 6.21(b), line 3. Change 5.3.11 to 5.3.12

• p. 230, Exercise 6.23. Brian Rothbach has pointed out that this problem can be given another stipulation: serieshelpmate in 20. By parity considerations (Black cannot lose an odd number of moves with a knight) the solution is the same as before except Black plays Pa7-a6 instead of Pa7-a5. Thus the number of solutions is $C_{10} = 16796$.

• p. 233, Exercise 6.30, line 3. It would be less ambiguous to change “this exercise” to “that exercise”.

• p. 235, Exercise 6.34, line 4. At the end of the line it should be mentioned that the polynomial $g(L_n, q)$ of Exercise 3.71(f) is a further $q$-analogue of $C_n$. An additional reference for this polynomial is R. Stanley, *J. Amer. Math. Soc.* 5 (1992), 805–851 (Prop. 8.6).

• p. 236, Exercise 6.34(b,c). While (b) is correct as stated (in the paperback edition of 2001), it would be best to change $q^n c_n(q)$ on line 6 to $c_n(q)$ (as it was in the hardcover edition of 1999) and “nonnegative” on line 8 to “nonpositive”. In this way part (c) remains valid. If part (b) is kept as it is, then change $c_n(t; q)$ in line 4 of part (c) to $q^n c_n(t; q)$.

• p. 238, Exercise 6.38(d), line 1. Change $(n, n)$ to $(n, 0)$.

• p. 239, Exercise 6.39(h). A period is missing at the end of the sentence.

• p. 241, Exercise 6.41, line 1. Change $S^2(w) = w$ to $S^2(w) = 12 \cdots n$.  

• p. 247, Exercise 6.59. See the item above for p. 192, lines 8– to 7–.

• p. 250, Exercise 6.3, line 3. Replace “$r = s + \frac{1}{2}$ for some $s \in \mathbb{Z}$” with “$r$ cannot be a negative integer”.

• p. 250, Exercise 6.3, paragraph 3. The earliest proof that $\sum_{n \geq 0} \left( \frac{2^n}{n} \right) t^t x^n$ isn’t algebraic for any $t \in \mathbb{N}$, $t > 1$, appears in the paper P. Flajolet, *Theoretical Computer Science* 49 (1987), 283–309 (page 294). Flajolet
shows that if $\sum a_n x^n$ is algebraic and each $a_n \in \mathbb{Q}$, then $a_n$ satisfies an asymptotic formula

$$a_n = \frac{\beta^n n^s}{\Gamma(s+1)} \sum_{i=0}^{m} C_i \omega_i^n + O(\beta^n n^t),$$

where $s \in \mathbb{Q} - \{-1, -2, -3, \ldots\}$, $t < s$, $\beta$ is a positive algebraic number, and the $C_i$ and $\omega_i$ are algebraic with $|\omega_i| = 1$. A simple application of Stirling’s formula shows that if $a_n = \left(\begin{array}{c} 2n \\ n \end{array}\right) t$, then $a_n$ does not have this asymptotic form when $t \in \mathbb{N}$, $t > 1$.

- p. 257, Exercise 6.19(k). Update the reference to *J. Integer Seq.* 4 (2001), Article 01.1.3; available electronically at

  \[ \text{http://www.research.att.com/~njas/sequences/JIS} \]

- p. 258, Exercise 6.19(s), line 1. Change $a_i$ to $a_i - 1$.


- p. 269, Exercise 4.23. Change the rating to [5].

- p. 274, line 2. Change “D. Vanquelin” to “B. Vauquelin”.

- p. 278, Exercise 6.53, line 3. Change $Q(x) = x - 2$ to $Q(x) = -x - 2$.


- p. 282, Exercise 6.63(b), line 2. Change 1847 to 1848.

- p. 293, lines 11-13. Replace “, and such that the ... exist.)” with a period. (The deleted condition automatically holds.)

- p. 295, Figure 7-3. In the expansion of $h_{41}$, the coefficient of $m_{41}$ should be 2.
• p. 298, line 10–. Change “if follows” to “it follows”.

• p. 301, line 7. Change 1.1.9(b) to 1.9(b).

• pp. 314–315, proof of Proposition 7.10.4. Change λ to λ/µ throughout proof.

• p. 317, line 12–. Change “clearly impossible” to “clear”.

• p. 329, line 15–. Change x’s to X’s.

• p. 336, line 7 (counting the displayed tableau as a single line). Change 7.8.2(b) to 7.8.2(a).

• p. 346, line 3–. Change “forms a border strip” to “forms a border strip or is empty”.

• p. 346, line 1–. Change λi/λi+1 to λi+1/λi.

• p. 348, line 9. Change χλ(µ) to χλ(µ).

• p. 352, line 2 of proof of Proposition 7.18.1. Change \( \sum_{\mu} z^{-1}_{\lambda} f(\lambda)p_{\mu} \) to \( \sum_{\lambda} z^{-1}_{\lambda} f(\lambda)p_{\lambda} \).

• p. 354, line 5. Change “a integral” to “an integral”.

• p. 355, line 4. Add a period after “nonnegative”.

• p. 359, line 6. Change the subscript \( \alpha_S \) to \( co(S) \).

• p. 364, line 1. Change \( e(D(T)) \) to \( e(co(D(T))) \).

• p. 370, line 3 of second proof. Change 1.22(d) to 1.23(d).

• p. 377, line 7; p. 378, line 8; page 378, line 10–. Change \( \pi \in B(r, c, t) \) to \( \pi \subseteq B(r, c, t) \).

• p. 379, line 5–. Insert \( \pi \) after the first “partition”, and change \( \lambda^* \) to \( \pi^* \).

• p. 379, line 4–. Change “similary” to “similarly”. 
• p. 383, line 9. Change “\(D(w) = T'\) and \(D(w^{-1}) = T\)” to “\(D(w) = D(T')\) and \(D(w^{-1}) = D(T)\)”.

• p. 395, line 10–. Change “Burnside’s theorem” to “Burnside’s lemma”.


• p. 416, line 7–. Change \(u_{i+2}\) to \(u_{j+2}\).

• p. 418, line 7. Change “subsequences” to “subsequence”.

• p. 419, line 16. Change “was” to “is”.

• p. 421, line 9–. Insert “a” after “such”.

• p. 421, lines 8– to 7–. Change “second statement of Theorem A1.1.4” to “first assertion of Theorem A1.1.6”.


• p. 424, line 11. Delete “by”.

• p. 426, line “tableaux in (A1.137)” to “tableau defined by (A1.137)”.

• p. 429, line 7. Delete comma after 156.

• p. 442, Theorem A2.4, line 9. Change “Hence” to “Moreover,”.

• p. 443, line 11. Change

\[
\text{char } \varphi = (x_1 \cdots x_n)^{-1} = (x_1 \cdots x_n)^{-1}s_\emptyset
\]

to

\[
\text{char } \varphi = x_1^{-1} + \cdots + x_n^{-1} = (x_1 \cdots x_n)^{-1}s_\emptyset^{n-1}
\]

• p. 444, line 12. Delete “char”.

• p. 444, line 11–. Change “given by (A2.156)” to “generated (as a \(\mathbb{C}\)-algebra) by (A2.156)”.

• p. 447, line 3–. Change \(s_1(x_1^{\lambda_i})\) to \(s_1(x_1^{\lambda_i}, x_2^{\lambda_i}, \ldots)\).
• p. 451, Exercise 7.13(a). This exercise is stated incorrectly. For instance, $K_{777,6654} = 1$, contrary to the statement of the exercise. One way to state the correct result is as follows. Let the parts of $\lambda'$ be given by as follows. Let the parts of $\lambda'$ be given by

$$\lambda'_1 = \cdots = \lambda'_{n_1} > \lambda'_{n_1+1} = \cdots = \lambda'_{n_2} > \lambda'_{n_2+1} = \cdots > \lambda'_{n_{k-1}+1} = \cdots = \lambda'_{n_k} > 0.$$  

Define $\lambda^{(j)} = (\lambda'_{n_{j-1}+1}, \ldots, \lambda'_{n_j})$ (with $n_0 = 0$), so $\lambda^{(j)}$ is a partition of rectangular shape. Let $\mu$ be a partition with $|\mu| = |\lambda|$, and let 

$$\mu^{(j)} = (\mu_{n_{j-1}+1}, \ldots, \mu_{n_j}).$$  

Then $K_{\lambda \mu} = 1$ if and only if $\lambda \geq \mu$ (dominance order) and 

(i) $|\lambda^{(j)}| = |\mu^{(j)}|$ and $\lambda^{(j)} \geq \mu^{(j)}$ for all $j$.

(ii) For all $1 \leq j \leq k$ either $0 \leq \mu'_{n_{j-1}+1} - \lambda'_{n_{j-1}+1} \leq 1$ or $0 \leq \lambda'_{n_j} - \mu'_{n_j} \leq 1$.

• p. 452, line 6. Change “$k$ times” to “$n$ times”.

• p. 452, Exercise 7/16(a), line 5. Change $c_{i-j} + c_{i+j}$ to Change $c_{i-j}i c_{i+j}$.

• pp. 452–453, Exercise 7.16(b,e). The formulas for $y_i(n)$ and $u_i(n)$ have been extended to $i \leq 6$ by F. Gascon, Fonctions de Bessel et combinatoire, Publ. LACIM 28, Univ. du Québec à Montréal, 2002 (page 75). In particular,

$$y_6(2n) = 6(2n)! \sum_{k=0}^{n} \frac{(10n - 13k + 8)C_{k+1}}{(n-k+2)! (n-k)! (k+4)! k!},$$

where $C_{k+1}$ denotes a Catalan number.

• p. 459, Exercise 7.30(c), line 4. Change $d - 1$ to $d$.

• p. 460, Exercise 7.37. For further information on expanding $a^2$ in terms of Schur functions, see

• p. 461, Exercise 7.42, line 2. Change $s \tilde{\lambda}(y)$ to $s \tilde{\lambda}'(y)$.

• p. 466, line 3–. Change $(\lambda_i - 1)!(\lambda'_i - 1)!$ to $(\lambda_i - i)! (\lambda'_i - i)!$.

• p. 467, Exercise 7.55(b). Let $f(n)$ be the number of $\lambda \vdash n$ satisfying (7.177). Then

$$(f(1), f(2), \ldots, f(30)) = (1, 1, 1, 2, 2, 7, 7, 10, 10, 34, 40, 53, 61, 103, 112, 143, 145, 369, 458, 579, 712, 938, 1127, 1383, 1638, 2308, 2754, 3334, 3925, 5092).$$

The problem of finding a formula for $f(n)$ was solved by Arvind Ayyer, Amritanshu Prasad, and Steven Spallone, arXiv:1604.08837.

• p. 467, Exercise 7.59. In order for the bijection $\lambda \mapsto (\lambda^0, \lambda^1, \ldots, \lambda^{p-1})$ given in the solution to part (e) (page 517) to be correct, it is necessary to define a specific indexing of the terms of $C_{\lambda}$. Namely, index a term $a$ by $c_i$ if $i = i_1 - i_0$, where $i_1$ is the number of 1’s weakly to the left of $a$, and $i_0$ is the number of 0’s strictly to the right of $a$ (so if $a = 1$, then this contributes to $i_1$). The sequence becomes $\cdots c_{-2} c_{-1} c_0 c_1 c_2 \cdots$ as before, so it suffices to define the indexing by letting the first 1 be $c_{i_{10}-1}$, where $i_0$ is the number of 0’s following this 1.

Example. If $\lambda = (4, 3, 3, 3, 1)$, then $C_{\lambda} = \cdots 0010110001011 \cdots$. The first 1 in this sequence is $c_{-4} = c_{-3}$. On the other hand, if $\lambda = (3, 3, 3, 2, 2, 1)$, then $C_{\lambda} = \cdots 0010100100011 \cdots$. Now the first 1 is $c_{1-6} = c_{-5}$.

• p. 468, Exercise 7.59(e), line 3. Change $Y^k$ to $Y^p$.

• p. 469, Exercise 7.61, line 2. Change “0 or 1” to “0 or ±1”.

• p. 474, Exercise 7.70, line 3. Under the second summation sign insert a space between “in” and $\mathfrak{S}_n$.

• p. 485, line 3–. The asymptotic formula for $a(n)$ should be multiplied by a factor of $1/\sqrt{3\pi}$. The factor $1/\sqrt{\pi}$ was included by Wright and omitted here by mistake. The additional factor $1/\sqrt{3}$ was omitted by Wright, though his proof makes it clear that it should appear. See L. Mutafchiev and E. Kamenov, math.CO/0601253.

• p. 491, Exercise 7.9, line 1. Insert $\varepsilon_{\lambda}$ before $a_{\lambda \mu} e_{\lambda}$.
• p. 492, Exercise 7.11. Change \( \binom{\ell}{j} \) to \( \binom{\ell(\mu)-1}{j} \) (three times).


• p. 494, Figure 7-20. Change the labels \( R_{16}, R_{15}, \) and \( R_{26} \) to \( R_{1a6}, R_{1a5}, \) and \( R_{2a5}, \) respectively.

• p. 496, equation (7.199). Change \( (m_i(\lambda)!)^{-1} \) to \( \prod_i (m_i(\lambda)!)^{-1} \).


• p. 500, displayed tableaux near end of Exercise 7.24. The tableaux \( T_8 \) and \( T_9 \) are missing the element 8 to the right of 3. Also, the \( \{3, 10\} \) under \( T_9 \) should be under \( T_{10} \).

• p. 502, Exercise 7.27, first displayed equation. Change \( (n)_m \) to \( (n)_{n-m} \).


• p. 514, line 1–, and p. 515, line 1. “Ibid.” refers to the reference in the item above, not to the previous reference in the book.


• p. 516, line 8. Change \((\lambda_i - 1)!^2(\lambda'_i - 1)! \) to \((\lambda_i - i)!^2(\lambda'_i - i)! \).

• p. 516, Exercise 7.54. The following elegant solution is due to Katherine Kalampogia-Evangelinou. Expand \(s_\lambda\) in terms of power sums and set \(x_i = q^{i-1}\) (principal specialization). If \(\mu\) has no even part, then \(p_\mu(1,q,q^2,\ldots)\) has no pole at \(q = -1\). If \(\lambda\) has an even hook length, then by Corollary 7.21.3 \(s_\lambda(1,q,q^2,\ldots)\) has a pole at \(q = -1\), and the proof follows.

• p. 517, Exercise 7.59(e), line 3. Change \(Y_k\) to \(Y^p\).

• p. 517, Exercise 7.59(e), line 9. Change \(Y_\emptyset\) to \(Y^p_\emptyset\), and change \(Y^k\) to \(Y^p\).

• p. 518, Exercise 7.59(h), line 1. Change \(Y_\emptyset\) to \(Y^p_\emptyset\), and change \(Y^k\) to \(Y^p\).

• p. 518, Exercise 7.59(h), line 2. Change \(Y^k\) to \(Y^p\) (three times).

• p. 518, Exercise 7.59(h), line 3. Change \(Y^k\) to \(Y^p\).

• p. 520, line 3. Change \(\sum_{n \geq 0} h_{2n+1}t^{2n+1}\) to \(\sum_{n \geq 0} (-1)^n h_{2n+1}t^{2n+1}\).

• p. 535, lines 7–10. Replace the sentence “No proof ... are known.” with “A bijective proof of the unimodality of \(s_\lambda(1,q,\ldots,q^n)\) was given by A. N. Kirillov, C. R. Acad. Sci. Paris, Sér. I 315 (1992), 497–501.”

• p. 537, Exercise 7.78(f), line 6. Change \(s_\mu(x)\) to \(s_\mu(y)\) and \(s_\nu(x)\) to \(s_\nu(z)\).

• p. 576, line 7. Change work to word.


• p. 580. Change “Valquelin, D.” to “Vauquelin, B.”.