

# EC2 SUPPLEMENT: ORIGINAL EDITION OF 1999

Richard P. Stanley

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Here I will maintain supplementary material for *Enumerative Combinatorics*, volume 2 (original edition of 1999). This will include errata, updated references, and new material. I will be continually updating this supplement.

**NOTE.** References to math.CO refer to the combinatorics section of the Mathematics Archive at [arxiv.org/list/math.CO/recent](http://arxiv.org/list/math.CO/recent). A front end site for math.CO is [front.math.ucdavis.edu/math.CO](http://front.math.ucdavis.edu/math.CO).

- p. 2, Example 5.1.2. Interchange  $\cap$  and  $\cup$  on line 2.
- p. 6, line 10. Change situations to situations.
- p. 8, line 6. The first  $\Pi$  should be  $\mathbf{\Pi}$ .
- p. 11, line 3. Change  $E_c(n)$  to  $E_c(x)$ .
- p. 18, line 3. Change  $(n)_2$  to  $n(n-2)$ .
- p. 20, line 9. Change  $Z(\mathfrak{S}_n)$  to  $\tilde{Z}(\mathfrak{S}_n)$ .
- p. 24, line 4 (after figure). Change  $\lim_{n \rightarrow \infty}$  to  $\lim_{k \rightarrow \infty}$ .
- p. 25, line 5. Change  $\subseteq$  to  $\in$ .
- p. 33, line 5-. Change  $\text{ord}(\tau_k)$  to  $\text{ord}(\tau_j)$ .
- p. 34, Lemma 5.3.9. Delete the first sentence, viz., “Let  $w \in \mathcal{A}^*$ .”
- p. 35, line 10. Change  $w \in \mathcal{B}^*$  to  $w \in \mathcal{B}_r^*$ .
- p. 35, line 8-. Change  $\mathbb{A}$  to  $\mathcal{A}$ .
- p. 36, lines 15–16. Change “beginning with a 1” to “ending with a  $-1$ ”.

- p. 36, line 1–. Insert  $+\cdots$  before  $=$ . (The left-hand side is an infinite sum.)
- p. 51, line 9–. Change  $Q_i = \Pi_i^{(2)}$  to “when  $Q_i$  is given by Example 5.5.2(d) for  $r = 2$ ”.
- p. 59, line 8. Change “effect” to “affect”.
- p. 59, line 9. Change “Since the rows” to “Since the columns”.
- p. 59, line 13. Change “Because the columns” to “Because the rows”.
- p. 62. Example 5.6.12, line 5. Change “modulo  $n$ ” to “modulo  $2^n$ ”.
- p. 63, line 12. Change “sequence” to “sequences”.
- p. 65, line 8. Change “Theorem” to “Lemma”.
- p. 72, Exercise 5.2(a). Relabel the first part (iii) as part (ii).
- p. 74, Exercise 5.8(a). The stated formula for  $T(n, k)$  fails for  $n = 0$ . Also, it makes more sense to define  $T(0, 0) = 1$ .
- p. 81, Exercise 5.24(d). A solution was found by the Cambridge Combinatorics and Coffee Club (February 2000).
- p. 83, line 1–. Change diagraph to digraph.
- p. 87, equation (5.111). We need to add the further condition that  $p_n(0) = \delta_{0n}$ . Otherwise, for instance, the polynomials  $p_n(x) = (1+x)^n$  satisfy (iv) with  $Q = \frac{d}{dx}$  but fail to satisfy (i)–(iii).
- p. 97, Exercise 5.51. It is not true that (ii) implies (i), e.g., when  $C(x) = c$ . One needs to add the hypothesis that  $[x]C(x) \neq 0$ , so that  $(C(x) - c)^{\langle -1 \rangle}$  exists. Substituting  $xC(B(x))$  for  $x$  in (ii) yields

$$xC(B(x))/C(A(xC(B(x)))) = x,$$

so  $C(B(x)) = C(A(xC(B(x))))$ . Substituting  $B(x)^{\langle -1 \rangle}$  for  $x$  yields  $C(x) = C(A(B(x)^{\langle -1 \rangle}C(x)))$ . Subtract  $c$  from both sides and apply  $(C - c)^{\langle -1 \rangle}$  to get  $x = A(B(x)^{\langle -1 \rangle}C(x))$ . Applying  $A^{\langle -1 \rangle}$  to both sides gives (i). This argument is due to Daniel Giaimo and Amit Khetan and (independently) to Yumi Odama.

- p. 101, line 3. Change  $J_0[(2-t)/\sqrt{t-1}]$  to  $J_0(\sqrt{-t}(2-t)/(1-t))$ .
- p. 102, Exercise 5.71. It would be better not to specify the degree  $d$  of  $G$ , since (as stated in the solution)  $d = \lambda_1$ .
- p. 103, Exercise 5.74(d). Replace the first two sentences with: Show that all vertices have the same outdegree  $d$ . (By (c), all vertices then also have indegree  $d$ .)
- p. 103, Exercise 5.74(f). For further information, see F. Curtis, J. Drew, C.-K. Li and D. Prigel, *J. Combinatorial Theory (A)* **105** (2004), 35–50, and the references given there.
- p. 124, Exercise 5.28. A bijective proof based on Prüfer codes is due to the Cambridge Combinatorics and Coffee Club (December 1999).
- p. 124, Exercise 5.29(b). Update the Pitman reference to *J. Combinatorial Theory (A)* **85** (1999), 165–193. Further results on  $P_n$  and related posets are given by D. N. Kozlov, *J. Combinatorial Theory (A)* **88** (1999), 112–122.
- p. 134, Exercise 5.41(c), lines 3– to 2–. The paper of Postnikov and Stanley has appeared in *J. Combinatorial Theory (A)* **91** (2000), 544–597.
- p. 136, last line of Exercise 5.41(j). A solution different from the one above was given by S. C. Locke, *Amer. Math. Monthly* **106** (1999), 168.
- p. 137, Exercise 5.45, line 1. Change  $kxy^k$  to  $(k+1)xy^k$ .
- p. 137, Exercise 5.45, line 4. Change this equation to
 
$$y = x + 2xy + 3xy^2 + \cdots = \frac{x}{(1-y)^2}.$$
- p. 139, Exercise 5.47(c), line 7. A direct combinatorial proof was given by M. Bousquet-Mélou and G. Schaeffer, *Advances in Applied Math.* **24** (2000), 337–368.
- p. 142, line 1. Change  $L^{n-1}$  to  $L^n$ .

- p. 143, Exercise 5.50(c), lines 3– to 2–. The paper of Postnikov and Stanley has appeared in *J. Combinatorial Theory (A)* **91** (2000), 544–597.
- p. 144, Exercise 5.53. The identity

$$4^n = \sum_{j=0}^n 2^{n-j} \binom{n+j}{j} \quad (1)$$

follows immediately from “Banach’s match box problem,” an account of which appears for instance in W. Feller, *An Introduction to Probability Theory and Its Applications*, vol. 1, second ed., Wiley, New York, 1957 (§5.8). This yields a simple bijective proof of (1).

- p. 147, Third Solution. The first two lines should be: Equation (5.530) can be rewritten (after substituting  $n+k$  for  $n$ )

$$(n+k)[x^n] \frac{1}{k} \left( \frac{F^{(-1)}(x)}{x} \right)^k = [x^n] \left( \frac{x}{F(x)} \right)^{n+k}. \quad (5.140)$$

- p. 151, Exercise 5.62(b). David Callan observed (private communication) that there is a very simple combinatorial proof. Any matrix of the type being enumerated can be written *uniquely* in the form  $P+2Q$ , where  $P$  and  $Q$  are permutation matrices. Conversely  $P+2Q$  is always of the type being enumerated, whence  $f_3(n) = n!^2$ .
- p. 162, lines 13– to 12–. Change “Thus any algebraic power series, as defined in Definition 6.1.1” to “Thus any algebraic function, i.e., any solution  $\eta$  to (6.2)”.
- p. 169, item (vi). When there is a region with only two edges, then the neighboring regions will not be convex (as shown in Figure 6.1). Hence when there is a region with two edges the phrase “each a convex  $k$ -gon” should be replaced by “each a  $k$ -gon”.
- p. 175, line 1. Change  $\{9, 11\}$  to  $\{9, 14\}$ .
- p. 175, line 2. Change  $x^{11}$  to  $x^{14}$ .
- p. 175, line 4. Change  $v^{11}$  to  $v^{14}$ .

- p. 175, line 11. Change Theorem 5.4.1 to Theorem 5.4.2.
- p. 175, line 2-. Change  $k \in K$  to  $k \in \mathbb{Z}$ .
- p. 176, line 16. Change intesect to intersect.
- p. 176, line 4-. Change  $(n + 2)$ -gon to  $(n + 1)$ -gon.
- p. 192, line 9-. Change  $u(0) = 0$  to  $v(0) = 0$ .
- p. 192, lines 8- to 7-. The example  $v = \log(1 + x^2) - 1$  is confusing since  $v(0) \neq 0$ . Nevertheless the series  $u(v(x)) = \sqrt{\log(1 + x^2)}$  is well-defined formally since we can write

$$\sqrt{\log(1 + x^2)} = x \sqrt{\frac{\log(1 + x^2)}{x^2}}.$$

It would have been more accurate to define

$$v(x) = \frac{\log(1 + x^2)}{x^2} - 1.$$

The same remarks apply to Exercise 6.59.

- p. 212, line 1. The statement that Catalan number enumeration originated with Segner and Euler in 1760 (or actually 1758/59 in the cited references) is inaccurate. The enumeration of polygon dissections was stated by Euler in a letter to Goldbach in 1751. This letter is printed in P.-H. Fuss, *Correspondance Mathématique et Physique*, Tome. 1, Acad. Sci. St. Petersburg, 1843; reprinted in *The Sources of Science*, No. 35, Johnson Reprint Corporation, New York and London, 1968, pp. 549–552.
- p. 212. For further details on the history of Catalan numbers, see P. J. Larcombe and P. D. C. Wilson, *Mathematics Today* **34** (1998), 114–117; P. J. Larcombe, *Mathematics Today* **35** (1999), 25, 89; P. J. Larcombe, *Math. Spectrum* **32** (1999/2000), 5–7; and P. J. Larcombe and P. D. C. Wilson, *Congr. Numerantium* **149** (2001), 97–108.
- p. 212, lines 16–17. I have forgotten the source for the statement that Netto was the first to use the term “Catalan number.” Can anyone provide a reference?

- p. 213, line 5–. Change “to Comtet [19]” “to Abel [continue??]” see Ouvres, vol II, p. 287, point D
- p. 217, Exercise 6.2(a). It needs to be assumed that  $F(0) = 0$ ; otherwise e.g.  $F(x) = 1/2$  is a trivial counterexample.
- p. 219, Exercise 6.16. A combinatorial proof was first given by R. Sulanke, *Electronic J. Combinatorics* **7**(1), R40, 2000. A sharper result was subsequently proved combinatorially by D. Callan, A uniformly distributed parameter on a class of lattice paths, math.CO/0310461.
- p. 221, Exercise 6.19(j). This problem appeared as Problem A5 on the 2003 William Lowell Putnam Mathematical Competition. Many participants found the following bijection with 6.19(i) (Dyck paths from  $(0, 0)$  to  $(2n, 0)$ ): Let  $D$  be a Dyck path from  $(0, 0)$  to  $(2n, 0)$ . If  $D$  has no maximal sequence of  $(1, -1)$  steps of even length ending on the  $x$ -axis, then just prepend the steps  $(1, 1)$  and  $(1, -1)$  to the beginning of  $D$ . Otherwise let  $R$  be the rightmost maximal sequence of  $(1, -1)$  steps of even length ending on the  $x$ -axis. Insert an extra  $(1, 1)$  step at the beginning of  $D$  and a  $(1, -1)$  step after  $R$ . This gives the desired bijection.
- p. 224, item **ii**, line 5. Change  $S(w) = w$  to  $S(w) = 12 \cdots n$ .
- p. 228, item **iii**, line 3. To be precise, the displayed sequences should have the initial and final 1's deleted.
- p. 230, Exercise 6.21(b), line 3. Change 5.3.11 to 5.3.12.
- p. 230, Exercise 6.23. Brian Rothbach has pointed out that this problem can be given another stipulation: serieshelpmate in 20. By parity considerations (Black cannot lose an odd number of moves with a knight) the solution is the same as before except Black plays Pa7-a6 instead of Pa7-a5. Thus the number of solutions is  $C_{10} = 16796$ .
- p. 231, Exercise 6.25(i). This conjecture has been proved by M. Haiman, *J. Amer. Math. Soc.* **14** (2001), 941–1006; math.AG/0010246.
- p. 232, Exercise 6.27(c). Robin Chapman has found an elegant argument that there always exists an integral orthonormal basis.

- p. 233, Exercise 6.30, line 3. It would be less ambiguous to change “this exercise” to “that exercise”.
- p. 235, Exercise 6.34, line 4. At the end of the line it should be mentioned that the polynomial  $g(L_n, q)$  of Exercise 3.71(f) is a further  $q$ -analogue of  $C_n$ . An additional reference for this polynomial is R. Stanley, *J. Amer. Math. Soc.* **5** (1992), 805–851 (Prop. 8.6).
- p. 236, Exercise 6.34(b), line 8. Change “nonnegative” to “nonpositive”.
- p. 238, Exercise 6.38(d), line 1. Change  $(n, n)$  to  $(n, 0)$ .
- p. 239, Exercise 6.39(h). A period is missing at the end of the sentence.
- p. 241, Exercise 6.41, line 1. Change  $S^2(w) = w$  to  $S^2(w) = 12 \cdots n$ .
- p. 246, Exercise 6.55(a), line 4. Change “while  $w(t) \geq i + 1$  if  $t$  is between  $k_i + 1$  and  $s$ ” to “while  $w(t) \geq i + 1$  if  $k_i + 1 \leq t \leq s$  or  $s \leq t \leq k_i - 1$ ”.
- p. 246, equation (6.62). Change  $\sum_{k=1}^{n-1}$  to  $\sum_{k=1}^n$ .
- p. 247, Exercise 6.59. See the item above for p. 192, lines 8– to 7–.
- p. 250, Exercise 6.3, line 3. Replace “ $r = s + \frac{1}{2}$  for some  $s \in \mathbb{Z}$ ” with “ $r$  cannot be a negative integer”.
- p. 250, Exercise 6.3, paragraph 3. The earliest proof that  $\sum_{n \geq 0} \binom{2n}{n} x^n$  isn’t algebraic for any  $t \in \mathbb{N}$ ,  $t > 1$ , appears in the paper P. Flajolet, *Theoretical Computer Science* **49** (1987), 283–309 (page 294). Flajolet shows that if  $\sum a_n x^n$  is algebraic and each  $a_n \in \mathbb{Q}$ , then  $a_n$  satisfies an asymptotic formula

$$a_n = \frac{\beta^n n^s}{\Gamma(s+1)} \sum_{i=0}^m C_i \omega_i^n + O(\beta^n n^t),$$

where  $s \in \mathbb{Q} - \{-1, -2, -3, \dots\}$ ,  $t < s$ ,  $\beta$  is a positive algebraic number, and the  $C_i$  and  $\omega_i$  are algebraic with  $|\omega_i| = 1$ . A simple application of Stirling’s formula shows that if  $a_n = \binom{2n}{n}^t$ , then  $a_n$  does not have this asymptotic form when  $t \in \mathbb{N}$ ,  $t > 1$ .

- p. 250, Exercise 6.4. A complete description of a field of generalized power series that forms an algebraic closure of  $\mathbb{F}_p[[x]]$  is given by K. S. Kedlaya, *Proc. Amer. Math. Soc.* **129** (2001), 3461–3470.
- p. 253, last two lines. Change “somewhat general more result” to “somewhat more general result”.
- p. 257, Exercise 6.19(k). Update the reference to *J. Integer Seq.* **4** (2001), Article 01.1.3; available electronically at

<http://www.research.att.com/~njas/sequences/JIS>.

- p. 258, Exercise 6.19(s), line 1. Change  $a_i$  to  $a_i - 1$ .
- p. 260, line 6–. Change  $(c_{j_\ell} + j_\ell - 1, n)$  to  $(n, j_\ell)$ .
- p. 260, Exercise 6.19(ee), lines 10–12. The statement that the first published proof of the enumeration of 321-avoiding permutations is due to D. G. Rogers is inaccurate. Knuth provided such a proof in 1973 in the reference given in the first paragraph of the solution. Moreover, a bijective proof was found by D. Rotem, *Inf. Proc. Letters* **4** (1975), 58–61.
- pp. 261–262, Exercise 3.19(pp). A further reference on noncrossing partitions is the nice survey article R. Simion, *Discrete Math.* **217** (2000), 367–409.
- p. 264, Exercise 6.19(iii). It should be mentioned that the diagonals of the frieze patterns of Exercise 6.19(mmm) are precisely the sequences  $1a_1a_2 \cdots a_n1$  of the present exercise.
- p. 265, Exercise 6.19(lll), lines 3– to 2–. The paper of Postnikov and Stanley has appeared in *J. Combinatorial Theory (A)* **91** (2000), 544–597.
- p. 265, Exercise 6.19(mmm). A couple of additional references to frieze patterns are H. S. M. Coxeter, *Acta Arith.* **18** (1971), 297–310, and H. S. M. Coxeter and J. F. Rigby, in *The Lighter Side of Mathematics* (R. K. Guy and R. E. Woodrow, eds.), Mathematical Association of America, Washington, DC, 1994, pp. 15–27.

- p. 269, line 1–, to p. 270, line 1. The paper of Postnikov and Stanley has appeared in *J. Combinatorial Theory (A)* **91** (2000), 544–597.
- p. 272, end of Exercise 6.33(c). Yet another proof was given by J. H. Przytycki and A. S. Sikora, *J. Combinatorial Theory(A)* **92** (2000), 68–76, math.CO/9811086.
- p. 272, Exercise 6.34, line 7. Change **a** to **e**.
- p. 274, line 2. Change “D. Vanquelin” to “B. Vauquelin”.
- p. 278, Exercise 6.53, line 3. Change  $Q(x) = x - 2$  to  $Q(x) = -x - 2$ .
- p. 279, Exercise 6.56(c). In the paper N. Alon and E. Friedgut, *J. Combinatorial Theory (A)* **89** (2000), 133–140, it is shown that  $A_v(n) < c^{n\gamma^*(n)}$ , where  $\gamma^*(n)$  is an extremely slow growing function related to the Ackermann hierarchy. The paper is available at

<http://www.ma.huji.ac.il/~ehudf>.

- p. 281, Exercise 6.60. An elegant proof based on Gröbner bases was given by Chris Hillar, *Proc. Amer. Math. Soc.* **132** (2004), 2693–2701.
- p. 291, line 9–. In general it is not true that  $\hat{\Lambda}_R = \hat{\Lambda} \otimes R$ ; one only has a natural surjection from the former onto the latter. Equality will hold if  $R$  is noetherian.
- p. 292, line 7. Insert “in” after “role”.
- p. 293, lines 11-13. Replace “, and such that the ... exist.)” with a period. (The deleted condition automatically holds.)
- p. 295, Figure 7-3. In the expansion of  $h_{41}$ , the coefficient of  $m_{41}$  should be 2.
- p. 298, line 10–. Change “if follows” to “it follows”.
- p. 300, line 8–. Change  $\sum$  to  $\prod$ .
- p. 301, line 7. Change 1.1.9(b) to 1.9(b).
- pp. 314–315, proof of Proposition 7.10.4. Change  $\lambda$  to  $\lambda/\mu$  throughout proof.

- p. 315, Figure 7-4. In the expression for  $s_3$  change the second  $m_{111}$  to  $m_3$ . Similarly, in the expression for  $s_4$  change the second  $m_{1111}$  to  $m_4$ .
- p. 317, line 12–. Change “clearly impossible” to “clear”.
- p. 322, line 2. Interchange  $\tilde{P}$  and  $\tilde{Q}$ .
- p. 326, line 2. Insert a space after “antichains”.
- p. 329, line 15–. Change  $x$ ’s to  $X$ ’s.
- p. 336, line 7 (counting the displayed tableau as a single line). Change 7.8.2(b) to 7.8.2(a).
- p. 346, line 3–. Change “forms a border strip” to “forms a border strip or is empty”.
- p. 346, line 1–. Change  $\lambda^i/\lambda^{i+1}$  to  $\lambda^{i+1}/\lambda^i$ .
- p. 348, line 9. Change  $\chi_\lambda(\mu)$  to  $\chi^\lambda(\mu)$ .
- p. 352, line 2 of proof of Proposition 7.18.1. Change  $\sum_{\mu} z_\lambda^{-1} f(\lambda) p_\mu$  to  $\sum_{\lambda} z_\lambda^{-1} f(\lambda) p_\lambda$ .
- p. 354, line 4. Change “in” to “is”.
- p. 354, line 5. Change “a integral” to “an integral”.
- p. 355, line 4. Add a period after “nonnegative”.
- p. 356, line 1. Insert “character of the” before “action”.
- p. 359, line 6. Change the subscript  $\alpha_S$  to  $\text{co}(S)$ .
- p. 364, line 1. Change  $e(D(T))$  to  $e(\text{co}(D(T)))$ .
- p. 370, line 3 of second proof. Change 1.22(d) to 1.23(d).
- p. 370, line 5–. Change the first row of the middle tableau from 43333311 to 4333311.

- p. 374, first diagram. The 1 at the end of the first row should be in boldface.
- p. 377, line 7; p. 378, line 8; page 378, line 10–. Change  $\pi \in B(r, c, t)$  to  $\pi \subseteq B(r, c, t)$ .
- p. 379, line 5–. Insert  $\pi$  after the first “partition”,
- p. 379, line 4–. Change “similary” to “similarly” and change  $\lambda^*$  to  $\pi^*$ .
- p. 381, middle of page. Replace  $\begin{array}{ccc} 0 & \mathbf{1} & \mathbf{1} \\ 0 & \mathbf{1} & \mathbf{1} \\ 0 & & \end{array}$  with  $\begin{array}{ccc} 0 & \mathbf{1} & \mathbf{1} \\ 0 & \mathbf{1} & \mathbf{1} \\ 0 & & \end{array}$ .
- p. 383, line 9. Change “ $D(w) = T'$  and  $D(w^{-1}) = T$ ” to “ $D(w) = D(T')$  and  $D(w^{-1}) = D(T)$ ”.
- p. 395, line 10–. Change “Burnside’s theorem” to “Burnside’s lemma”.
- p. 399, line 15. Change “function” to “functions”.
- p. 399, line 7–. For additional information concerning Craige Schensted, see the webpage <http://ea.ea.home.mindspring.com>.
- p. 404, line 7–. Change A2.2 to A2.4.
- p. 405, line 1. Change A2.6 to A2.8.
- p. 405, line 6. Change A2.6 to A2.8.
- p. 416, line 7–. Change  $u_{i+2}$  to  $u_{j+2}$ .
- p. 418, line 7. Change “subsequences” to “subsequence”.
- p. 419, line 16. Change “was” to “is”.
- p. 421, line 9–. Insert “a” after “such”.
- p. 421, lines 8– to 7–. Change “second statement of Theorem A1.1.4” to “first assertion of Theorem A1.1.6”.
- p. 422, line 3. Change A1.1.4 to A1.1.6.
- p. 424, line 11. Delete “by”.

- p. 426, line “tableaux in (A1.137)” to “tableau defined by (A1.137)”.
- p. 439, line 7. Delete comma after 156.
- p. 439, reference A1.13. An updated version of this paper of van Leeuwen, entitled “The Littlewood-Richardson rule, and related combinatorics,” is available at [math.CO/9908099](http://math.CO/9908099).
- p. 442, Theorem A2.4, line 6. change  $\alpha : V \rightarrow W$  to  $\alpha : W \rightarrow W'$ .
- p. 442, Theorem A2.4, line 7. Change  $v \in V$  to  $v \in W$ .
- p. 442, Theorem A2.4, line 9. Change “Hence” to “Moreover,”.
- p. 443, line 11. Change

$$\text{char } \varphi = (x_1 \cdots x_n)^{-1} = (x_1 \cdots x_n)^{-1} s_{\emptyset}$$

to

$$\text{char } \varphi = x_1^{-1} + \cdots + x_n^{-1} = (x_1 \cdots x_n)^{-1} s^{1^{n-1}}$$

- p. 444, line 12. Delete “char”.
- p. 444, line 11–. Change “given by (A2.156)” to “generated (as a  $\mathbb{C}$ -algebra) by (A2.156)”.
- p. 447, line 3–. Change  $s_1(x_1^{\lambda_i})$  to  $s_1(x_1^{\lambda_i}, x_2^{\lambda_i}, \dots)$ .
- p. 450, Exercise 7.4, line 2. Change the exponent  $n - 1 - r$  to  $n - 1 + r$ .
- p. 451, Exercise 7.13(a). This exercise is stated incorrectly. For instance,  $K_{777,6654} = 1$ , contrary to the statement of the exercise. One way to state the correct result is as follows. Let the parts of  $\lambda'$  be given by

$$\begin{aligned} \lambda'_1 = \cdots = \lambda'_{n_1} > \lambda'_{n_1+1} = \cdots = \lambda'_{n_2} > \lambda'_{n_2+1} = \cdots \\ > \lambda'_{n_{k-1}+1} = \cdots = \lambda'_{n_k} > 0. \end{aligned}$$

Define  $\lambda^{(j)} = (\lambda'_{n_{j-1}+1}, \dots, \lambda'_{n_j})$  (with  $n_0 = 0$ ), so  $\lambda^{(j)}$  is a partition of rectangular shape. Let  $\mu$  be a partition with  $|\mu| = |\lambda|$ , and let

$$\mu^{(j)} = (\mu_{n_{j-1}+1}, \dots, \mu_{n_j}).$$

Then  $K_{\lambda\mu} = 1$  if and only if  $\lambda \geq \mu$  (dominance order) and

- (i)  $|\lambda^{(j)}| = |\mu^{(j)}|$  and  $\lambda^{(j)} \geq \mu^{(j)}$  for all  $j$ .
- (ii) For all  $1 \leq j \leq k$  either  $0 \leq \mu'_{n_{j-1}+1} - \lambda'_{n_{j-1}+1} \leq 1$  or  $0 \leq \lambda'_{n_j} - \mu'_{n_j} \leq 1$ .

- p. 452, line 6. Change “ $k$  times” to “ $n$  times”.
- pp. 452–453, Exercise 7.16(b,e). The formulas for  $y_i(n)$  and  $u_i(n)$  have been extended to  $i \leq 6$  by F. Gascon, *Fonctions de Bessel et combinatoire*, Publ. LACIM **28**, Univ. du Québec à Montréal, 2002 (page 75). In particular,

$$y_6(2n) = 6(2n)! \sum_{k=0}^n \frac{(10n - 13k + 8)C_{k+1}}{(n - k + 2)!(n - k)!(k + 4)!k!},$$

where  $C_{k+1}$  denotes a Catalan number.

- p. 459, Exercise 7.30(b), line 2. Change  $x_i^{d-1} + x_i^{d-2}x_j + x_i^{d-3}x_j^2 + \cdots + x_j^{d-2}x_i^{d-1}$  to  $x_i^d + x_i^{d-1}x_j + x_i^{d-2}x_j^2 + \cdots + x_j^d$ .
- p. 459, Exercise 7.30(c), line 4. Change  $d - 1$  to  $d$ .
- p. 460, Exercise 7.37. For further information on expanding  $a_\delta^2$  in terms of Schur functions, see

<http://www.phys.uni.torun.pl/~bgw/vanex.html>.

- p. 461, Exercise 7.42, line 2. Change  $s_{\bar{\lambda}}(y)$  to  $s_{\bar{\lambda}'}(y)$ .
- p. 466, line 3–. Change  $(\lambda_i - 1)!(\lambda'_i - 1)!$  to  $(\lambda_i - i)!(\lambda'_i - i)!$ .
- p. 467, line 5. Change  $\mathfrak{S}_n$  to  $\mathfrak{S}_n$ .
- p. 467, Exercise 7.55(b). Let  $f(n)$  be the number of  $\lambda \vdash n$  satisfying (7.177). Then

$$(f(1), f(2), \dots, f(30)) = (1, 1, 1, 2, 2, 7, 7, 10, 10, 34, 40, 53, 61, 103, 112, 143, 145, 369, 458, 579, 712, 938, 1127, 1383, 1638, 2308, 2754, 3334, 3925, 5092).$$

- p. 467, Exercise 7.59. In order for the bijection  $\lambda \mapsto (\lambda^0, \lambda^1, \dots, \lambda^{p-1})$  given in the solution to part (e) (page 517) to be correct, it is necessary to define a specific indexing of the terms of  $C_\lambda$ . Namely, index a term  $a$  by  $c_i$  if  $i = i_1 - i_0$ , where  $i_1$  is the number of 1's weakly to the left of  $a$ , and  $i_0$  is the number of 0's strictly to the right of  $a$  (so if  $a = 1$ , then this contributes to  $i_1$ ). The sequence becomes  $\dots c_{-2}c_{-1}c_0c_1c_2\dots$  as before, so it suffices to define the indexing by letting the first 1 be  $c_{i_0-1}$ , where  $i_0$  is the number of 0's following this 1.

*Example.* If  $\lambda = (4, 3, 3, 3, 1)$ , then  $C_\lambda = \dots 0010110001011\dots$ . The first 1 in this sequence is  $c_{1-4} = c_{-3}$ . On the other hand, if  $\lambda = (3, 3, 3, 2, 2, 1)$ , then  $C_\lambda = \dots 0010100100011\dots$ . Now the first 1 is  $c_{1-6} = c_{-5}$ .

- p. 468, Exercise 7.59(e), line 3. Change  $Y^k$  to  $Y^p$ .
- p. 469, Exercise 7.61, line 2. Change “0 or 1” to “0 or  $\pm 1$ ”.
- p. 474, Exercise 7.70, line 3. Under the second summation sign insert a space between “in” and  $\mathfrak{S}_n$ .
- p. 477, Exercise 7.79(c), line 1. Change “strenghtening” to “strengthening”.
- p. 484, equation (7.193). Change  $1 \leq i \leq j \leq n$  to  $1 \leq i < j \leq n$ .
- p. 484, Exercise 7.101(b). As in (a), the plane partitions being counted have largest part at most  $m$ .
- p. 485, line 4. Change SSYT to “reverse SSYT” (i.e., the rows are weakly decreasing and columns strictly decreasing).
- p. 485, line 5. Change  $T_{ij} < n - \lambda_i + i$  to  $T_{ij} \leq n + \mu_i - i$ , and change  $n = 3$  to  $n = 2$ .
- p. 485, lines 6 and 8. Change  $t_{32/1,3}(q)$  to  $t_{32/1,2}(q)$ .
- p. 485, line 7. The five displayed tableaux should be rotated  $180^\circ$ .
- p. 485, line 3-. The asymptotic formula for  $a(n)$  should be multiplied by a factor of  $1/\sqrt{3\pi}$ . The factor  $1/\sqrt{\pi}$  was included by Wright and omitted here by mistake. The additional factor  $1/\sqrt{3}$  was omitted by

Wright, though his proof makes it clear that it should appear. See L. Mutafchiev and E. Kamenov, [math.CO/0601253](#).

- p. 491, Exercise 7.9, line 1. Insert  $\varepsilon_\lambda$  before  $a_{\lambda\mu}e_\lambda$ .
- p. 493, Exercise 7.13(a). For another proof, see A. N. Kirillov, *Europ. J. Combinatorics* **21** (2000), 1047–1055, arXiv:hep-th/9304099 (Prop. 2.2).
- p. 494, line 4. Change 169–172 to 175–177.
- p. 494, Figure 7-20. Change the labels  $R_1h6$ ,  $R_1h5$ , and  $R_2h6$  to  $R_1a6$ ,  $R_1a5$ , and  $R_2a5$ , respectively.
- p. 496, equation (7.199). Change  $(m_i(\lambda)!)^{-1}$  to  $[\prod_i(m_i(\lambda)!)^{-1}]$ .
- p. 497. Exercise 7.22(b), line 2. Change the first  $\mathfrak{R}_n$  to  $\mathfrak{S}_n$ .
- p. 498, Exercise 7.22(h), line 7. Update the Fomin and Greene reference to *Discrete Math.* **193** (1998), 179–200.
- p. 500, displayed tableaux near end of Exercise 7.24. The tableaux  $T_8$  and  $T_9$  are missing the element 8 to the right of 3. Also, the  $\{3, 10\}$  under  $T_9$  should be under  $T_{10}$ .
- p. 500, line 5–. Change (??) to (c).
- p. 502, Exercise 7.27, first displayed equation. Change  $(n)_m$  to  $(n)_{n-m}$ .
- p. 504, line 10–. Update the Babson, et al., reference to *Topology* **38** (1999), 271–299.
- p. 505, Exercise 7.32(a). Stembridge’s more general result appears in “Computational aspects of root systems, Coxeter groups, and Weyl characters,” in *Interactions of Combinatorics and Representation Theory*, MSJ Memoirs **11**, Math. Soc. Japan, Tokyo, 2001, pp. 1–38 (Theorem 7.4).
- p. 514, Exercise 7.47(m), lines 1–3. Update the reference to R. Stanley, *Discrete Math.* **193** (1998), 267–286.
- p. 514, Exercise 7.48(b), lines 2–4. Update the reference to R. Simion and R. Stanley, *Discrete Math.* **204** (1999), 369–396.

- p. 514, lines 4– and 3–. Change “*ibid.*, Cor. 7.1.2” to “R. Stanley, *Electron. J. Combinatorics* **3**, R6 (1996), 22 pp., Cor. 1.2”.
- p. 514, line 1–, and p. 515, line 1. “*Ibid.*” refers to the reference in the item above, not to the previous reference in the book.
- p. 515, line 3. “the reference” refers to the reference two items above, viz., R. Stanley, *Electron. J. Combinatorics* **3**, R6 (1996), 22 pp.
- p. 515, Exercise 7.48(g). Further generalizations of shuffle posets are considered by P. Hersh, *J. Combinatorial Theory (A)* **97** (2002), 1–26.
- p. 515, Exercise 7.49. Update this reference to C. Lenart, *J. Algebraic Combin.* **11** (2000), 69–78.
- p. 516, line 8. Change  $(\lambda_i - 1)!(\lambda'_i - 1)!$  to  $(\lambda_i - i)!(\lambda'_i - i)!$ .
- p. 517, Exercise 7.59(e), line 3. Change  $Y^k$  to  $Y^p$ .
- p. 517, Exercise 7.59(e), line 9. Change  $Y_\emptyset$  to  $Y_{p,\emptyset}$ , and change  $Y^k$  to  $Y^p$ .
- p. 518, Exercise 7.59(h), line 1. Change  $Y_\emptyset$  to  $Y_{p,\emptyset}$ , and change  $Y^k$  to  $Y^p$ .
- p. 518, Exercise 7.59(h), line 2. Change  $Y^k$  to  $Y^p$  (three times).
- p. 518, Exercise 7.59(h), line 3. Change  $Y^k$  to  $Y^p$ .
- p. 520, line 3–. Change  $\sum_{n \geq 0} h_{2n+1} t^{2n+1}$  to  $\sum_{n \geq 0} (-1)^n h_{2n+1} t^{2n+1}$
- p. 534, end of Exercise 7.74. For some connections between inner plethysm and graphical enumeration, see L. Travis, Ph.D. thesis, Brandeis University, 1999, math.CO/9811127.
- p. 535, lines 7–10. Replace the sentence “No proof . . . are known.” with “A bijective proof of the unimodality of  $s_\lambda(1, q, \dots, q^n)$  was given by A. N. Kirillov, *C. R. Acad. Sci. Paris, Sér. I* **315** (1992), 497–501.”
- p. 537, Exercise 7.78(f), line 6. Change  $s_\mu(x)$  to  $s_\mu(y)$  and  $s_\nu(x)$  to  $s_\nu(z)$ .

- p. 539, Exercise 7.85. A further reference to the evaluation of  $g_{\lambda\mu\nu}$  is M. H. Rosas, The Kronecker product of Schur functions indexed by two-row shapes or hook shapes, [math.CO/0001084](https://arxiv.org/abs/math.CO/0001084).
- p. 542, line 10. Update the Babson, et al., reference to *Topology* **38** (1999), 271–299.
- p. 544, lines 4– to 2–. Update the reference to R. Stanley, *Discrete Math.* **193** (1998), 267–286.
- p. 551, Exercise 7.102(b), lines 2– to 1–. The “nice” bijective proof asked for was given by M. Rubey, A nice bijection for a content formula for skew semistandard Young tableaux, [math.CO/0011099](https://arxiv.org/abs/math.CO/0011099). The proof is based on jeu de taquin.
- p. 554, last two lines of Exercise 7.107(a). Update reference to *Annals of Combinatorics* **2** (1998), 103–110.
- p. 556, line 3. Change  $n \rightarrow \infty$  to  $x \rightarrow \infty$ .
- p. 556, line 6. Change  $(x - t)^2$  to  $(x - t)$ .
- p. 556, line 7. Change  $n^{1/6}$  to  $n^{1/3}$ .
- p. 576, line 7. Change work to word.
- p. 580. Replace index entry “triangle-free graph” with “triangle-free graph”.
- p. 580. Change “Valquelin, D.” to “Vauquelin, B.”.