ERRATA
for *Catalan Numbers*
version of 14 January 2016

- p. 1, line 8. Change $d - 1$ to $n - 1$.
- p. 40, item 132. The five examples should be
  \[ 12132434 \quad 12134234 \quad 12314234 \quad 12312434 \quad 12341234 \]
- p. 51, item 200. The condition on $A$ and $B$ should be that for all $i$, the $i$th largest element of $A$ is smaller than the $i$th largest element of $B$.
- p. 134, Problem A59. It should be assumed in both parts that $f(x)$ has compact support; otherwise the solution is not unique.
ADDENDA

version of 14 January 2016

B1. (a) [2+] Define integers \( c_n \) by
\[
C(-x) = \prod_{n \geq 1} (1 - x^n)^{c_n}.
\]
Show that
\[
c_n = \frac{1}{2n} \sum_{d|n} (-1)^{d-1} \mu(n/d) \binom{2d}{d}.
\]
(b) [2+] Show that \( c_n \) is divisible by \( n \).
(c) [3–] Show that \( 6c_n \) is divisible by \( n^2 \).

B2. [3] Fix \( n \geq 2 \). Let \( X \) be a \((2n - 1)\)-element set. Let \( V \) be the real (any field of characteristic 0 will do) vector space with a basis consisting of all symbols
\[
[a_1, \ldots, a_i, [b_1, \ldots, b_n], a_{i+1}, \ldots, a_{n-1}],
\]
where \( \{a_1, \ldots, a_{n-1}, b_1, \ldots, b_n\} = X \). Let \( W \) be the subspace of \( V \) generated by the following elements:

- \([c_1, \ldots, c_i, c_{i+1}, \ldots, c_{2n-1}] + [c_1, \ldots, c_{i+1}, c_i, \ldots, c_{2n-1}] \). In other words, the \((2n - 1)\)-component “bracket” \([c_1, \ldots, c_{2n-1}] \) (where each \( c_i \) is an element of \( X \) with one exception which is a bracket \([b_1, \ldots, b_n] \) of elements of \( X \) is antisymmetric in its entries.

- For all \( a_1 < \cdots < a_{n-1} \) and \( b_1 < \cdots < b_n \) such that \( \{a_1, \ldots, a_{n-1}, b_1, \ldots, b_n\} = X \), the element
\[
[a_1, \ldots, a_{n-1}; [b_1, \ldots, b_n]] - \sum_{i=1}^{n} [b_1, \ldots, b_{i-1}, [a_1, \ldots, a_{n-1}, b_i], b_{i+1}, \ldots, b_n].
\]
Show that \( \dim V/W = C_n \).
Solutions

B1. (c) See http://mathoverflow.net/questions/195339.

B2. This result was conjectured by Tamar Friedmann (in a more general context) and proved by Phil Hanlon in 2015. Friedmann made the stronger conjecture that the natural $\mathfrak{S}_n$-action on $V/W$ is the irreducible representation indexed by the partition $(2, 2, \ldots, 2, 1)$ of $2n - 1$. Hanlon in fact proved this stronger conjecture.