ERRATA

for Catalan Numbers

version of 24 March 2016

• p. 1, line 8. Change $d - 1$ to $n - 1$.

• p. 40, item 132. The five examples should be

12132434 12134234 12314234 12312434 12341234

• p. 51, item 200. The condition on $A$ and $B$ should be that for all $i$, the $i$th largest element of $A$ is smaller than the $i$th largest element of $B$.

• p. 134, Problem A59. It should be assumed in both parts that $f(x)$ has compact support; otherwise the solution is not unique.

• p. 213, column 1, line 4. Change Martin to Michael.
ADDENDA

version of 15 July 2016

B1. (a) [2+] Define integers $c_n$ by

$$C(-x) = \prod_{n \geq 1} (1 - x^n)^{c_n}.$$ 

Show that

$$c_n = \frac{1}{2n} \sum_{d \mid n} (-1)^{d-1} \mu(n/d) \binom{2d}{d}.$$ 

(b) [2+] Show that $c_n$ is divisible by $n$.

(c) [3–] Show that $6c_n$ is divisible by $n^2$.

B2. [3] Fix $n \geq 2$. Let $X$ be a $(2n-1)$-element set. Let $V$ be the real (any field of characteristic 0 will do) vector space with a basis consisting of all symbols

$$[a_1, \ldots, a_i, [b_1, \ldots, b_n], a_{i+1}, \ldots, a_{n-1}],$$

where $\{a_1, \ldots, a_{n-1}, b_1, \ldots, b_n\} = X$. Let $W$ be the subspace of $V$ generated by the following elements:

• $[c_1, \ldots, c_i, c_{i+1}, \ldots, c_{2n-1}] + [c_1, \ldots, c_{i+1}, c_i, \ldots, c_{2n-1}]$. In other words, the $(2n-1)$-component “bracket” $[c_1, \ldots, c_{2n-1}]$ (where each $c_i$ is an element of $X$ with one exception which is a bracket $[b_1, \ldots, b_n]$ of elements of $X$) is antisymmetric in its entries.

• For all $a_1 < \cdots < a_{n-1}$ and $b_1 < \cdots < b_n$ such that $\{a_1, \ldots, a_{n-1}, b_1, \ldots, b_n\} = X$, the element

$$[a_1, \ldots, a_{n-1}, [b_1, \ldots, b_n]] - \sum_{i=1}^{n} [b_1, \ldots, b_{i-1}, [a_1, \ldots, a_{n-1}, b_i], b_{i+1}, \ldots, b_n].$$

Show that $\dim V/W = C_n$.

B3. [3–] Let $n$ denote the $n$-element chain $1 < 2 < \cdots < n$. Show that for $n \geq 3$, $C_n$ is the number of $n$-element subsets $S$ of the poset $n \times n$ with the following properties: (a) $S$ intersects every maximal chain of $n \times n$ and is minimal with respect to this property, (b) $S$ lies below the equator, i.e., if $(i,j) \in S$ then $i + j \leq n + 1$, and (c) $(n,1) \in S$. 
Solutions

B1. (c) See http://mathoverflow.net/questions/195339.

B2. This result was conjectured by Tamar Friedmann (in a more general context) and proved by Phil Hanlon in 2015. Friedmann made the stronger conjecture that the natural $\mathfrak{S}_n$-action on $V/W$ is the irreducible representation indexed by the partition $(2, 2, \ldots, 2, 1)$ of $2n-1$. Hanlon in fact proved this stronger conjecture.