18.466 Problem set 2, due Monday, March 4, 2013

The handout "Sufficiency," Theorem 3, shows that the order statistics $(X_{(1)}, ..., X_{(n)})$ are sufficient for the large family of all distributions P^n for all probability measures on the real line. To see that the order statistics are minimal sufficient, it will be enough to show that they are minimal sufficient when P is restricted to either of two families indexed by one-dimensional parameters θ, as follows.

1. Bickel and Doksum, p. 86, problem 10. (Each of x and θ ranges over the whole line.)

2. Bickel and Doksum, p. 87, problem 15. Again each of x and θ ranges over the whole line. *Hint*: consider the reciprocal of the likelihood function as a function of θ.

3. In the handout "Exponential families," in the example of the $N(\mu, \sigma^2)$ family, $C(\theta(\psi))$ is what it must be to normalize a probability density, and so must be a function of $\theta(\psi) = (\theta_1(\psi), \theta_2(\psi))$. Find the function explicitly.

4. Bickel and Doksum, pp. 87–88, Problem 4. *Hint*: parts (a), (b), and (c) are very easy, for the same reason.

5. Bickel and Doksum, p. 88, Problem 5. Find the natural parameter spaces for each of these families.