COMMENTS ON THE UNADJUSTED DIP TEST QUANTILE TABLE

The table "qDiptab" in the "diptest" package was generated as follows. Although unimodality is a highly composite hypothesis, the uniform $U[0,1]$ was used to generate quantiles as a "borderline" unimodal distribution in that it tends at least asymptotically for large n to produce empirical distributions F_n farther from unimodal than for any other unimodal F, according to a very plausible conjecture by Hartigan and Hartigan (1985). For each value of n listed, the following Monte Carlo experiment was done by Maechler (first reported in 2004, current version of documentation 2012, p. 7): $N = 1{,}000{,}001$ times, n i.i.d. $U[0,1]$ variables X_j were generated and the dip statistics of the samples of size n were taken. Let $D_{(j)}$ be the jth order statistic of these N dip statistics for $j = 1, ..., N$. For $0 < q < 1$ (where q as in the table indicates "quantile" or $1 - p$), let $j(Nq)$ be the nearest integer to Nq (there is no ambiguity since Nq can't be a half-integer for the given q and N). Then the number in the body of the table for the given n and q is $D_{(j(Nq))}$. For such a large N, the simulation was rather computer-intensive.

So, the quantiles are not exact but simulated. It may be hard to find error bounds for the quantiles, but we can find error bounds for q or for the p-value $p = 1 - q$. Given X observed successes in N independent trials with unknown probability p_0 of success, with $\hat{p} = X/N$, the endpoints of the 95% quadratic confidence interval for p_0 are the two solutions for p of $(p - \hat{p})^2 = 1.96^2 p(1 - p)/N$. These intervals are generally superior to the plug-in intervals, which have $\hat{p}$ rather than p on the right.

When $q = 0.95$ so $\hat{p} = 0.05$, a 95% confidence interval for the true value of p is $[0.049575, 0.050429]$ or $[0.05 - 4.25 \cdot 10^{-4}, 0.05 + 4.29 \cdot 10^{-4}]$.

When $q = 0.99$ so $\hat{p} = 0.01$, a 95% confidence interval for the true value of p is $[0.009807, 0.010197]$ or $[0.01 - 1.93 \cdot 10^{-4}, 0.01 + 1.97 \cdot 10^{-4}]$.

When $q = 0.999$ so $\hat{p} = 0.001$, a 95% confidence interval for the true value of p is $[0.00093994, 0.0010639]$ or $[0.001 - 6.01 \cdot 10^{-5}, 0.001 + 6.39 \cdot 10^{-5}]$.

These possible errors in the p-values seem no larger and perhaps smaller than one has in using an asymptotic approximation for the

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distribution of a statistic. Whereas, in the original paper of Hartigan and Hartigan (1985, p. 80), a table of quantiles is given for the same n's, based on a similar study, but with $N = 9999$, smaller by two orders of magnitude. The corresponding confidence intervals would be wider. The quantiles given in the two tables, when they are not equal to the minimal possible value $1/(2n)$, usually differ in the fourth decimal place and often also in the third.

REFERENCES

Hartigan, J. A., and Hartigan, P. M. (1985). The dip test of unimodality. *Ann. Statist.* **13**, 70-84. Maechler, Martin (2012). Package 'diptest.' cran.r-project.org/web/packages/diptest/diptest.pdf