8. $\eta : G^0_{1,\mathrm{iso}}(n) \longrightarrow \mathbb{C}$

is a global 'log-det' function

$d\eta$ is a closed 1-form on $A^0_{1,\mathrm{iso}}$

s.t. $\frac{1}{2\pi i} d\eta$ is integral $\to$ integrates to 'det'.

$$\begin{tikzcd}
G^{-\infty}_{1,\mathrm{iso}}(n) \arrow[r, hook] & G^0_{1,\mathrm{iso}}(n) \arrow[r] & A^0_{1,\mathrm{iso}}(n) \\
\mathbb{I} + \Psi^{-\infty}_{1,\mathrm{iso}}(n) \arrow[r, hook] & E^0_{1,\mathrm{iso}}(n) \arrow[u, phantom, "\cap"] \arrow[ur] &
\end{tikzcd}$$

Here $E^0_{1,\mathrm{iso}}(n)$ is the <u>preimage</u> of $A^0_{1,\mathrm{iso}}(n)$

$$P \in E^0_{1,\mathrm{iso}}(n) \iff \exists\, A \in \Psi^{-\infty}_{1,\mathrm{iso}}(n) \text{ s.t. } P + A \in G^0_{1,\mathrm{iso}}(n)$$

Perturbative invertibility.

<u>Th$^{\underline{m}}$</u>: $G^0_{1,\mathrm{iso}}(n) \hookrightarrow E^0_{1,\mathrm{iso}}(n)$ is <u>dense</u>

$P_0, P_1 \in G^0_{1,\mathrm{iso}}(n)$ are in same component iff

$\exists\, P_t : [0,1] \to E^0_{1,\mathrm{iso}}(n)$ s.t. $df(P_1, P_0) = 0$,