

Real blow ups
Introduction to Analysis on Singular Spaces
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ABSTRACT. These are notes based on (and at least in part for) three lectures on real blow up that were given as part of the Introductory workshop for the program at the Mathematical Sciences Research Institute on Analysis on (or is it of?) Singular Spaces, Fall 2008.

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Introduction

In these three lectures I want to discuss real blow up as it relates to resolution of singular spaces and other analytic objects, especially Lie algebras of vector fields. Since this is quite a large subject, and other people will talk on certain aspects of it, I will concentrate on the geometric part – the definition and properties of blow up. Otherwise, as far as things I will use and also applications, I will simply summarize.