INTRODUCTION TO MICRLOCAL ANALYSIS
18.157 SPRING 2000

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In the first half of the course I will define pseudodifferential operators and discuss their basic properties. To do this I will start with a description of the Fourier transform on tempered distributions on $\mathbb{R}^n$ and use this to develop the standard oscillatory integral representation of the kernels of pseudodifferential operators. After describing composition and invariance properties I will discuss the standard algebra of pseudodifferential operators on a compact manifold and the fundamental 'symbol sequence':

$$0 \longrightarrow \Psi^{-1}(X) \longrightarrow \Psi^0(X) \xrightarrow{\sigma} \mathcal{C}^\infty(S^*X) \longrightarrow 0.$$

In the second part of the semester I will discuss some at least of the following related global results:

Spectrum of elliptic operators.
Hodge (and deRham) theorem.
Traces, star products and homology.

$$\mathrm{HH}_k(\Psi^{\mathbb{Z}}(X))/\Psi^{-\infty}(X)) \simeq H^{2n-k}(S^*X \times \mathbb{S}).$$

Determinants and the determinant bundle.
Eta invariants.
Adiabatic limits.
Relations to K-theory.

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