**18.02 Practice Problems for Final**

The following are a selection of problems whose solutions are in the Notes; after each group of problems is the page number (S.xx) of their solutions. You can pick the topics you feel you need most review on.

I would suggest reviewing by looking first at the four hour exams, and the practice hour exams—redo those problems, paying attention to the ones you originally missed. You have solutions to all of these. Then use the problems on this sheet for additional practice.

**Final Exam:** Will have about 20 short problems, covering the term's work evenly except there will be some extra weighting on Stokes' theorem.

1. Simple vector proofs; using the dot product formula:

$$A \cdot B = |A||B|\cos\theta = a_1 b_1 + a_2 b_2 + a_3 c_3\,.$$

12.1/53   12.2/43, 62   794/18 (S.1)

2. Vector cross product: its direction and magnitude and the determinantal formula for it. Equations of lines and planes.

   12.3/7, 14, 19 (S.2, 3)   12.4/7, 32, 37 (S.5)

3. Matrices: matrix multiplication, calculating $\det A$ and $A^{-1}$ for $2 \times 2$ and $3 \times 3$ matrices. Solving $Ax = b$ by using $A^{-1}$ and by Cramer's rue.

   A square system $Ax = 0$ has a non-trivial solution $\Leftrightarrow \det A = 0$.

   L.17/18a   L.18/4   L.19/1b, 3a (S.4)

4. Parametric equations. Finding the velocity, acceleration, unit tangent vector and speed of a motion.

   601/40a (S.6)   12.5/11, 39 (S.7)

5. Tangent plane to $z = f(x, y)$; tangent linear approximation:

$$\delta w \approx f_x \Delta x + f_y \Delta y\,.$$

13.4/39, 57 (S.9)   TA–7/3, 5 (S.10)

6. Directional derivative $dw/ds$; gradient $\nabla f$ (including geometric interpretations of its direction and magnitude); deducing approximate values of $dw/ds$ and $\nabla f$ from map of the level curves of $f(x, y)$.

   $\nabla f$ is normal vector to contour surfaces of $f(x, y, z)$; from this one gets the tangent plane to $f(x, y, z) = c$. Contour curves.

   13.8/12, 23, 32   (S.13)

7. Chain rule, when variables are independent and when not independent.

   13.7/5, 43, 49 (S.12)   N.3/3a, 4 (S.16)

8. Finding maxima and minima: with and without Lagrange multipliers. Application to line-fitting by method of least squares.

13.5/40, 46 (S.11) 13.9/24 (S.14) LS/1 (S.13)

9. Double integrals; putting in limits, changing the order of integration.
   I.1/1, 2 (S.19) 14.2/15, 30, 37 (S.20)
10. Double integrals in polar coordinates.
    I.2/5 (S.21) 14.4/2, 16, 23 14.5/33 956/36 (S.22)
11. Triple integrals in rectangular and cylindrical coordinates.
    14.6/5, 33 (S.23) 14.7/9, 12 (S.24) I.3/11, 12 (I.5)
12. Spherical coordinates; gravitational attraction.
    I.4/14, 16 (I.5) 14.7/26 (S.24) G.4/3 (S.25)
13. Line integrals; path independence, conservative fields (in the plane) and exact differentials. Finding the (mathematical) potential function.
    15.2/11, 32, 36 (S.26) Notes 2.7/1, 5, 6 (S.27, 8, 9)
14. Green's theorem, work form: SP.4/4–C1(b), 4–C4 (S.30)
    flux form: Notes 3.4/4 (S.32) 4.5/4a (S.33)
15. Surface integrals and flux: Notes 9.8/4, 5, 8 (S.36, 37) (Basic surfaces: sphere, cylinder and the graph of $z = f(x, y)$.)
16. Divergence theorem.
    Notes 10.5/5, 6, 8 (S.38, 39)
17. Line integrals in 3-space and conservative fields; exact differentials; finding the potential function.
    Notes 11.5/1, 4 12.5/2, 3 (S.40, 41)
18. Stokes' theorem, curl $F$; del operator $\nabla$, its use in formulas.
    SP.7/5-B4 (S.43) 15.7/1, 2 (S.46)