18.02, Problem set III. Due Feb 18, 12:45 in 2-106

If you haven't done so already, remember to join the 1802 list. There is an in-class test Friday, Feb 19 – which is why this problem set is due on Thursday, Feb 18. Material in the lecture on Thursday Feb 18 will *not* be included in the test – so it will be based on the first 6 lectures, sections 10.4, 12.1-12.6 of Edwards and Penney and parts L and K of the supplementary notes.

Problems are from the text (EP=Edwards and Penney) or from the supplementary notes (SN).

Hand in underlined problems from Part I and all of Part II. Note that the solutions to Part I problems are generally available in the notes, Section S. Part II is marked more critically with points as indicated. These points will accumulate and finally constitute 30% of the possible total.

Part I (1 pt each for correct answers)

Lec 5 (Thurs Feb 11): Read EP Sect 12.4, 10.4 to p. 591, 12.5 to p. 747. Problems:EP p. 742 nos. 3, 7, 22, 33. EP p. 594, nos. 4, 12, 15. EP p. 755 nos. 3, 4, 13, 31, 39, 40.
Lec 6 (Fri Feb 12): Read EP Sect 12.6, SN K. Problems SN p. K.2, nos. 1, 2, 3.

Part II

Problem 1: (3 pts)

Given that $\mathbf{A} = \mathbf{2i} + \mathbf{j} + \mathbf{3k}$ and $\mathbf{B} = \mathbf{3i} - \mathbf{2j} - \mathbf{k}$ find all vectors of length 4 which are perpendicular to both $\mathbf{A}$ and $\mathbf{B}$.

Solution: The cross product is $\mathbf{A} \times \mathbf{B} = (-1+6)\mathbf{i} - (-2-9)\mathbf{j} + (-4-3)\mathbf{k} = 5\mathbf{i} + 11\mathbf{j} - 7\mathbf{k}$. This has length $\sqrt{195}$ so the two vectors of length 4 perpendicular to both $\mathbf{A}$ and $\mathbf{B}$ are

$$\pm\frac{2}{\sqrt{195}}(5\mathbf{i} + 11\mathbf{j} - 7\mathbf{k}).$$

Problem 2: (2+2 pts)

Let O be the origin, $P = (2, 0, 1)$ and let L be the line through the origin parallel to $2\mathbf{i} - \mathbf{j} + \mathbf{2k}$.

a) Express the vector $\mathbf{OP}$ as the sum of a vector parallel to L and a vector perpendicular to L.

b) Calculate the distance from P to L.

Solution:

a) $\mathbf{OP} = 2\mathbf{i} + \mathbf{k}$ and $\mathbf{A} = 2\mathbf{i} - \mathbf{j} + \mathbf{2k}$ is parallel to the line. Since $\mathbf{A}$ has length 3 the vector $\frac{1}{9}(\mathbf{OP} \cdot \mathbf{A})\mathbf{A} = \frac{2}{3}\mathbf{A}$ has the same dot product with $\mathbf{A}$ as $\mathbf{OP}$ has (namely 6.) Thus

$$\mathbf{OP} = \frac{2}{3}(2\mathbf{i} - \mathbf{j} + \mathbf{2k}) + \mathbf{\frac{1}{3}}(\mathbf{2i} + \mathbf{2j} - \mathbf{k})$$

is the (only) decomposition into a part parallel to the line and a part perpendicular to it.

b) A general point on the line is $\frac{1}{3}(2\mathbf{i}+2\mathbf{j}-\mathbf{k})+t\mathbf{A}$ (since it is $\mathbf{OP}$ plus some multiple of $\mathbf{A}$). The length of this is greater than or equal to $|\frac{1}{3}(2\mathbf{i}+2\mathbf{j}-\mathbf{k})|=1$ which is therefore the distance from the origin to the line.

Problem 3: (2+2 pts)

Consider the system $x_1+x_2=2cx_1$, $x_1+2x_2+x_3=2cx_2$, $x_2+x_3=2cx_3$.

a) For what values of the constant c will there be a non-trivial solution?

b) Let $c=0$. Find a non-trivial solution by writing the three equations in vector form as $\mathbf{A}\cdot\mathbf{x}=0$, $\mathbf{B}\cdot\mathbf{x}=0$, and $\mathbf{C}\cdot\mathbf{x}=0$, then using vector analysis to find a non-zero vector $\mathbf{x}$ which is orthogonal to all three vectors $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$.

Solution:

a) The determinant of the matrix $\begin{bmatrix}(1-2c) & 1 & 0\\ 1 & (2-2c) & 1\\ 0 & 1 & (1-2c)\end{bmatrix}$ is

$$2(1-c)(1-2c)^2-2(1-2c)=2(1-2c)(2c^2-3c+1-1)=2c(1-2c)(2c-3).$$

Thus the determinant vanishes, and hence there is a non-trivial solution, if $c=0$, $c=\frac{1}{2}$ or $c=\frac{3}{2}$.

b) If $c=0$ then the equations are $\mathbf{A}\cdot\mathbf{x}=0$, $\mathbf{B}\cdot\mathbf{x}=0$, and $\mathbf{C}\cdot\mathbf{x}=0$, if $\mathbf{A}=\mathbf{i}+\mathbf{j}$, $\mathbf{B}=\mathbf{i}+2\mathbf{j}+\mathbf{k}$ and $\mathbf{C}=\mathbf{j}+\mathbf{k}$. The cross product of $\mathbf{A}$ and $\mathbf{C}$ is $\mathbf{i}-\mathbf{j}+\mathbf{k}$ – which is orthogonal to all three. Thus a non-trivial solution is $x_1=1$, $x_2=-1$ and $x_3=1$.

Problem 4: (8 pts)

Does a pitched “curve ball” in baseball really curve? Work through project 12.5 in EP. There a several questions to be answered; in your answer label then (a), (b) and so on, in order. Write up the last two in decent English.