18.02, Problem set II. Due Feb 12, 12:45 in 2-106

Remember to join the 1802 list, so you can get announcements – such as that this problem set is available!

General rules for homework (I will not keep on repeating this mantra, but it continues to apply): Please try each problem by yourself for 20 minutes or so. After this you may collaborate. However if you do so you are required to write the name of the person (or persons) with whom you have collaborated on your homework. It is not permissible to copy solutions from other people, nor to consult solutions from previous years.

Problems are from the text (EP=Edwards and Penney) or from the supplementary notes (SN) you need to get a copy of these too! Only the version of the notes dated at the bottom of the front page 'M.I.T. 1998' will do. Older versions have different problems.

Hand in underlined problems from Part I and all of Part II. You will receive 1 point for each problem in Part I that you hand in – whether correct or not – provided you have attempted it and not copied from someone else, etc. Note that the solutions to Part I problems are generally available in the notes, Section S. Part II is marked more critically with points as indicated. Remember that these points will accumulate and finally constitute 30% of the possible total.

Part I

Lec 2: Determinants and cross product. Read EP Sect 12.3, SN (pages) L.1-L.4top. Problems:SN L16 1-1, <u>2</u>, 3, <u>5a</u>, <u>6</u>, <u>7</u>, 9. (Solns SN S.2.) EP p.734 nos. 3, 7, 11, <u>14</u>, 16, <u>19</u>. (Solns SN S.3.)

Lec 3: Matrices and inversion. Read SN L.4 - L.11. Problems SN p. L.17 nos. <u>2-5b</u>, <u>2-8a</u>. SN p. L.18 <u>3-3</u>, <u>3-4</u>, <u>3-5</u> (Solns SN S.3 S.4).

Lec 4: Cramer's rule and square linear systems. Read SN L.12 - L.15. Problems SN p. L.19/L.20 <u>4-1b</u>, <u>4-3abc</u>, <u>4-7</u> (Solns SN S.5).

Part II

Problem 1: (2 pts, after Feb 2)

Derive the formula $\cos(\alpha-\beta) = \cos(\alpha)\cos(\beta)+\sin(\alpha)\sin(\beta)$ for $\alpha > \beta$ between 0 and $\pi/2$ by interpreting the right-hand side as the scalar product of two unit vectors.

Problem 2: (3=1+2 pts, after Feb 2)

1. Show that the four points $(-1,-1,-1)$, $(1,1,-1)$, $(-1,1,1)$ and $(1,-1,1)$ form the vertices of regular tetrahedron (state your reasoning precisely – a regular tetrahedron is one with all sides of equal length).

2. Find the angle in radians between two sides at any one vertex (use a calculator or computer) to two decimal places.

Problem 3: (3=1+1+2+2 pts, after Feb 4)

Given the vectors $\mathbf{A} = \mathbf{6\hat{i} - 2\hat{j} + 3\hat{k}}$ and $\mathbf{B} = \mathbf{3\hat{i} + 6\hat{j} - 2\hat{k}}$

1. Show that **A** and **B** are perpendicular (orthogonal).
2. Find unit vectors $\hat{i}'$ and $\hat{j}'$ in the directions of **A** and **B**.
3. Using the cross product find a third *unit* vector $\hat{k}'$ which is orthogonal to both $\hat{i}'$ and $\hat{j}'$ and such that $\hat{i}', \hat{j}', \hat{k}$ is a right-handed coordinate system.
4. If $\mathbf{C} = 2\hat{i} - \hat{j} + \hat{k}$ find constants a, b, c such that $\mathbf{C} = a\hat{i}' + b\hat{j}' + c\hat{k}'$ (hint – use the dot product).

Problem 4: (3 pts, after Feb 4)

Consider a tetrahedron with one vertex at the origin and each of the other three vertices on the three coordinate axes, say at the points $\langle a, 0, 0\rangle$, $\langle 0, b, 0\rangle$ and $\langle 0, 0, c\rangle$. Let D be the *area* of the side which is not in any one of the coordinate planes and let A, B and C be the areas of the other three sides (which are all right-angled triangles). Show that

$$D^2 = A^2 + B^2 + C^2$$

which is a three-dimensional form of Pythagoras's theorem.

Problem 5: (3 pts, after Feb 5)

A certain wafer manufactoring company makes three colored products by adding dyes to a sugar base (yuck). The dyes are Red (R), Blue (B) and Yellow (Y). On one day there are three production runs, with the three varieties using the following quantities, in ounces, of each dye per 100 pounds:

Type1:R=2,B=7,Y=2

Type2:R=6,B=1,Y=2

Type3:R=0,B=5,Y=3

What would you call the products? No, seriously, the total amounts of each dye used that day (when the Feds came to investigate) was 238 ounces of Red, 98 ounces of Blue and 162 ounces of Yellow. How much of Type3 wafer was produced that day?