ERRATA FOR “RATIONAL POINTS ON VARIETIES”

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This is an errata list for the book


• Definition 1.3.1: “direct product” should be “finite direct product”.
• Remark 3.5.62: “Theorem 3.5.59” should be “Definition 3.5.59”.
• Section 3.5.15: The sentence “But $X'_R$ need not be smooth.” is correct, but it would be more to the point to say “But $X'_R$ need not be regular.”
• Proof of Theorem 5.3.1: The actions were not specified clearly. The left translation action on $G$ induces a right $G$-action on $A$, which can be turned into a left $G$-action on $A$ (in which $g$ acts as right action of $g^{-1}$). It is this left $G$-action on $A$ and the induced contragredient left $G$-action on $A^*$ that are used in Step 2.
• The references to [WIT10] at the beginning of Section 5.7.2 and in the proof of Theorem 5.7.13 should be to [WIT08]. (Thank to Borys Kadets for pointing this out.)
• Section 5.11: Alex Youcis pointed out that the terminology needs to be fixed to reflect standard usage, which is as follows. There are maps

$$H^1(k, G) \to H^1(k, \text{Inn} G_{k_s}) \to H^1(k, \text{Aut} G_{k_s}),$$

where $\text{Inn} G_{k_s}$ is the group of inner automorphisms of $G(k_s)$. The algebraic groups corresponding to elements in the image of the second map (resp. the composition) are called inner forms (resp. pure inner forms), or sometimes inner twists (resp. pure inner twists). Some authors make a distinction between “inner twist” and “inner form”, using the former to mean an element of $H^1(k, \text{Inn} G_{k_s})$ and the latter to mean the image of such an element in $H^1(k, \text{Aut} G_{k_s})$ or the corresponding algebraic group.
• Thanks to the proof of the purity conjecture [Čes19], some simplifications are possible:
  - In Theorem 6.8.3, the “caveat” can be simplified to “the caveat that one must exclude the $p$-primary part of all the groups if there exists $x \in X^{(1)}$ such that $k(x)$ is imperfect of characteristic $p$”.
  - In Corollaries 6.8.5 and 6.8.7, the caveats are unnecessary.
  - In the proof of Proposition 6.9.10, the char $k = 0$ proof then works in arbitrary characteristic.
• Warning 6.8.4: $Br_k(X)$ should be $Br k(X)$.
• Proof of Lemma 6.9.8: Where Theorem 6.9.7 is invoked, Corollary 6.7.8 should be mentioned too.
• Theorem 8.4.10 and Corollary 8.4.11: Fei Xu pointed out that it is necessary to add the hypothesis “If $\text{char} k = p$, assume that $X$ is proper.” In the proof of Theorem 8.4.10, change the sentence starting “For any nonarchimedean $v \in S$” to

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“For any nonarchimedean \( v \in S \), there are only finitely many possibilities for the \( k_v \)-scheme \( F^{-1}(x_v) \) as \( x_v \) ranges over \( X(k_v) \): when \( \text{char } k = 0 \), this follows since \( k_v \) has only finitely many extensions of each degree; when \( \text{char } k = p \), use Krasner’s lemma (Proposition 3.5.74) and compactness of \( X(k_v) \).”

- **Remark 8.4.12**: Change “irrational” to “rational”, and “dominant morphism \( \mathbb{P}^1 \to X \)” to “morphism \( \mathbb{P}^1 \to X \) inducing a surjection \( \mathbb{P}^1(A^S) \to X(A^S) \)”.

- **Section A.4**: Juan Climent Vidal pointed out that \( \{x, \{y\}\} \) does not necessarily determine \( x \) and \( y \), and that it should be changed to Kuratowski’s definition \( \{\{x\}, \{x, y\}\} \).

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REFERENCES


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