Solve 30 points worth of problems for an A.

1. (5 points) The following program performs QR decomposition of an \( n \times n \) matrix. What is the exact total number of floating point operations?

```matlab
function A = house(A);
    n = size(A,1);
    for i = 1:n-1
        u = A(i:n,i);
        u(1) = u(1) + sign(u(1)) * norm(u);
        u = u / norm(u);
        A(i:n,i:n) = A(i:n,i:n) - 2 * u * (u' * A(i:n,i:n));
    end
```

Assume that \( \text{norm}() \) is computed using \( \text{norm}(x) = \sqrt{x_1^2 + \cdots + x_n^2} \).

2. (5 points) Show that every real matrix has a real SVD.

3. (5 points) Explain the choice of sign in the Householder reflectors, i.e. \( u = x + \text{sign}(x_1) \|x\|e_1 \) instead of \( u = x - \text{sign}(x_1) \|x\|e_1 \).

4. (5 points) Let \( A \) be real symmetric and positive semidefinite, i.e. \( x^T A x \geq 0 \) for all \( x \neq 0 \). Show that if the diagonal of \( A \) is zero, then \( A \) is zero.

5. A matrix \( A \) is called strictly column diagonally dominant if \( |a_{ii}| > \sum_{j \neq i} |a_{ji}| \) for all \( i \).

   (a) (5 points) Show that such an \( A \) is nonsingular.

   (b) (5 points) Show that no pivoting is needed when computing \( A = LU \). In other words, if we did do partial pivoting to compute \( PA = LU \), \( P \) a permutation matrix, then we would get \( P = I \). Hint: Show that after one step of Gaussian elimination, the bottom right \( n-1 \) by \( n-1 \) submatrix is also strictly column diagonally dominant.

6. (5 points) Prove or disprove the following statement: If \( A \) is a square matrix and \( \| \cdot \| \) is any matrix norm, then \( \| A \| < 1 \) implies that \( I - A \) is invertible.

7. (5 points) Consider evaluating the following expression in IEEE decimal floating point arithmetic, where every rounding (if it occurs) is toward 0, rather than toward the nearest floating point number. Prove or disprove that \( \text{fl}(((4./3.) - 1.) * 3.) - 1.) < 0 \).