The Usual Instructions

- Write up your solutions neatly, preferably with all pages stapled. You need not show every arithmetic calculation, but must always show enough work to demonstrate the process by which the answer is reached. Without this, the grader can’t be sure that you didn’t just copy the answers from someone else, and there’s no way to give partial credit.
- You’re free to work together in groups, but you must write up the solutions independently. Plagiarism is easy to detect.

Reading


Ungraded problems

Do the following exercises for practice—preferably after the corresponding lecture—but do not hand them in. The solutions are available to you, so you should check your work. Starred problems are especially recommended.

Each problem is from the Notes unless stated otherwise:

- 5A-1, 5A-3, 5A-5*, 5C-2 to 5C-7, 5C-9, 5C-11*
- 5D-1 to 5D-8
- 5D-10 to 5D-14, 5E-1, 5E-10a, 5E-11*

Graded problems, Part A [58 pts total]

*From Simmons:*

- 9.5 #2, 6, 10 [2 pts each], 14 [4 pts]
- 9.7 #26 [3 pts], 27 (use implicit differentiation) [3 pts]
- 10.3 #2, 6, 12, [3 pts each], 26 [5 pts]
- 10.4 #2, 4, 8, 18, 20 [4 pts each]
- 10.5 #10, 16 [4 pts each]

Graded problems, Part B [13 pts total]

1. Consider the indefinite integral \( \int \frac{dx}{x^2 - 1} \).

   (a) [3 pts] Compute it by using the fact that \( \frac{1}{x^2 - 1} = \frac{1/2}{x-1} + \frac{-1/2}{x+1} \).

   (b) [3 pts] Compute it by trigonometric substitution. (You might find it helpful to look up \( \int \sec u \, du \) or \( \int \csc u \, du \) on the inside back cover of Simmons.)

   (c) [2 pts] If both answers are correct, they need not be equal, but should differ only by a constant. Verify that this is the case.
2. [5 pts] Consider the following calculation: we want to compute \( \int_{-2}^{2} \frac{dx}{\sqrt{x^2 - 1}} \). Substituting \( x = \sec \theta \), we have \( dx = \sec \theta \tan \theta \, d\theta \) and \( \sqrt{x^2 - 1} = \sqrt{\sec^2 \theta - 1} = \tan \theta \), so

\[
\int \frac{dx}{\sqrt{x^2 - 1}} = \int \frac{\sec \theta \tan \theta \, d\theta}{\tan \theta} = \int \sec \theta \, d\theta = \ln |\sec \theta + \tan \theta| = \ln \left| x + \sqrt{x^2 - 1} \right|.
\]

To be sure we’ve done this right, we can check that \( \frac{d}{dx} \ln \left| x + \sqrt{x^2 - 1} \right| = \frac{1}{\sqrt{x^2 - 1}} \). Hence

\[
\int_{-2}^{2} \frac{dx}{\sqrt{x^2 - 1}} = \left[ \ln \left| x + \sqrt{x^2 - 1} \right| \right]_{-2}^{2} = \ln \left| 2 + \sqrt{3} \right| - \ln \left| -2 + \sqrt{3} \right| = \ln \left( \frac{2 + \sqrt{3}}{2 - \sqrt{3}} \right).
\]

This answer is utter nonsense. Explain why.