1. Show that if \( f : X \to Y \) is a fibration, and \( Y \) is based, then the canonical map

\[
f^{-1}(* ) \to F(f)
\]

is a homotopy equivalence.

2. A Serre fibration is a map \( f : X \to Y \) satisfying a restricted form of the homotopy lifting property. For all \( n \geq 0 \) and all \( g, h \) making the outer square commute

\[
\begin{array}{ccc}
I^n \times \{0\} & \xrightarrow{g} & X \\
\downarrow & & \downarrow f \\
I^{n+1} & \xrightarrow{h} & Y
\end{array}
\]

there exists a dotted arrow as above making the diagram commute. The notion of Serre fibration is often times more convenient than the notion of fibration.

Suppose that \( Y \) is pointed. Show that the canonical map \( f^{-1}(* ) \to F(f) \) is a weak equivalence. Deduce that Serre fibrations have long exact sequences of homotopy groups.

3. (Path-loop fibration) Let \( X \) be a pointed space.

(a) Show that the evaluation map

\[
ev_1 : \text{Map} (I, X) \to X
\]

is a Serre fibration, with fiber \( \Omega X \). (Note: it is actually a fibration.) This fiber sequence is called the path-loop fibration.

(b) Show that if \( p : E \to X \) is a Serre fibration with contractible total space \( E \), and fiber \( F \), then there is a weak equivalence \( F \to \Omega X \). (Hint: one approach is to compare with the LES of the path-loop fibration.)

4. Show that all locally trivial bundles are Serre fibrations.

5. Let \( H \) be a closed sub-Lie group of a compact Lie group \( G \). Show that \( G \to G/H \) is a locally trivial bundle with fiber \( H \). (Note: I think that the assumption that \( G \) is compact is not necessary, but might make the problem easier.)