# 18.02A Exam 1 Review - Spring 2007

There are several practice problems from each topic that may appear on the exam; you can expect approximately one problem from each of the numbered groups.

**1.** (*Changes of variables* )

**a)** Make the change of variables $u = x + y, v = x - y$ in order to evaluate the double integral

$$\int_0^1 \int_0^{1-x} (x+y)^3 \sin(x-y)\, dydx.$$

**b)** Let $R$ be the region bounded by the four lines $y = 2x, y = 2x + 2, y = -2x$, and $y = 2 - 2x$. Calculate

$$\iint_R (y^2 - 4x^2) e^{8x^2+2y^2}\, dA$$

by making the change of variables $u = y + 2x, v = y - 2x$.

**c)** Let $R$ be the region in the first octant $(x, y, z \geq 0)$ bounded by the surface $x + 2y + \left(\frac{x}{2}\right)^2 = 1$ and $z = \frac{2}{y}$. Rewrite the integral

$$\iiint_R \frac{1}{xy}\, dV$$

in terms of the variables $u = x + 2y, v = \frac{x}{2}$ and $w = yz$, but do not evaluate it.

**2.** (*Triple integrals* )

**a)** Let $R$ be the region in the first octant that is bounded by the coordinate planes ($x = 0, y = 0, z = 0$) and also by the plane with $x$-intercept $a$, $y$-intercept $b$, and $z$-intercept $c$. Calculate $R$'s center of mass. *Hint: By symmetry, only one integral needs to be evaluated.*

**b)** Consider a cone of height $b$ whose base is a circle of radius $a$ in the $x$-$y$ plane, centered at the origin. Calculate the moment of inertia about the $z$-axis.

**c)** A sphere of radius $a$ has density function $\delta = \gamma|z|$. Calculate its total mass. Find the mass of a sphere with density function $x^2 + y^2$ as well.

**3.** (*Vector fields* )

**a)** Sketch the vector field $\mathbf{F} = |x|\,\hat{\imath} + |y|\,\hat{\jmath}$.

**b)** Sketch the vector field $\mathbf{F} = \frac{y}{x}\,\hat{\imath} - \hat{\jmath}$.

**c)** Sketch the vector field $\mathbf{F} = (x + y)\,\hat{\imath} + (y - x)\,\hat{\jmath}$.

**d)** Write the equation for the vector field that always points in the positive $\hat{\jmath}$ direction, with magnitude equal to the distance from the origin.

**e)** Write an equation for the vector field that is perpendicular to the ray from the origin and whose magnitude is half the distance from the origin.

**4.** (*Line integrals* )

**a)** If $\mathbf{F} = (y-x)\,\hat{\imath} - y^2\,\hat{\jmath}$ and $\mathbf{c}$ is the counterclockwise path around the points $(0,0), (1,0)$, and $(1,1)$, evaluate $\int_{\mathbf{c}} \mathbf{F} \cdot d\mathbf{r}$.

**b)** Let $\mathbf{F} = \hat{\imath} - \hat{\jmath}$. Argue geometrically to find $\int_{\mathbf{c}} \mathbf{F} \cdot d\mathbf{r}$ for the straight line paths from $(1,0)$ to $(3,0)$ and also from $(-1,1)$ to $(1,-1)$.

**c)** Calculate explicitly the integrals from part b).

**d)** Evaluate

$$\int_{\mathbf{c}} (x + \sin z)\,dx + (4 - x^2)\,dy + 3y\,dz,$$

where $\mathbf{c} = (\sin t, -\cos t, t)$ for $0 \leq t \leq 2\pi$.

**5.** (*Conservative/Path-independent/Gradient fields* )

**a)** In the following problems, let $\mathbf{F} = |y|\,\hat{\imath} - x^2\,\hat{\jmath}$.

i) Suppose that the path $\mathbf{c}$ follows the rectangle with vertices $(2,1), (2,-1), (-2,-1)$, and $(-2,1)$. Calculate $\oint_{\mathbf{c}} \mathbf{F} \cdot d\mathbf{r}$.

ii) Is this field conservative?

**b)** Find the value(s) of $a$ that make the following field conservative:

$$\mathbf{F} = \left(\frac{a}{x} - \frac{2y}{x^2}\right)\hat{\imath} - \frac{2}{x}\,\hat{\jmath}.$$

**c)** Let $\mathbf{F} = \vec{\nabla}(x^2 + \tan^{-1}(xy))$, and evaluate

$$\int_{(0,1)}^{(\sqrt{3},1)} \mathbf{F} \cdot d\mathbf{r}.$$

**d)** Evaluate

$$\int_{\mathbf{c}} \sin\left(\frac{\pi y}{2}\right)\cos\left(\frac{\pi x}{2}\right)\,dx + \sin\left(\frac{\pi x}{2}\right)\cos\left(\frac{\pi y}{2}\right)\,dy$$

along the path that follows the clockwise circle of radius 1 from $(0,-1)$ to $(1,0)$.

**6.** (*Potential functions* )

**a)** Find the potential function for

$$\mathbf{F} = (-3z\sin x - 4(x+y))\,\hat{\imath} - 4(x+y)\,\hat{\jmath} + 3\cos x\,\hat{\mathbf{k}}.$$

**b)** Find the value(s) of $a$ for which $\mathbf{F} = (x\cos ax - y^2)\,\hat{\imath} - 2xy\,\hat{\jmath}$ is a gradient field, and calculate the potential function(s).

**c)** Find the potential function for

$$\mathbf{F} = \frac{x}{(x^2+y^2)^b}\,\hat{\imath} + \frac{y}{(x^2+y^2)^b}\,\hat{\jmath},$$

where $b$ is any real value.