

(Pr Exam #2)

See comments
on Practice Exam #1.

18.01 Practice Exam 4

Problem 1. (15) Evaluate $\int_0^{1/2} \frac{x^2}{\sqrt{1-x^2}} dx$ by making a trigonometric substitution; remember to change the limits also.

Problem 2. (15) Find the volume of the solid obtained by rotating about the y -axis (that's the y -axis) the area under the graph of $y = e^x$ and over the interval $0 \leq x \leq 1$.
(Suggestion: use cylindrical shells.)

Problem 3. (20) Evaluate $\int \frac{4x}{(x^2-1)(x-1)} dx$.

(Begin by factoring the denominator completely; don't forget the final integration.)

Problem 4. (15: 8, 7)

a) Write down the definite integral of the form $\int_a^b f(x) dx$ which represents the arclength of the curve $y = \sin^2 x$ for $0 \leq x \leq \pi/4$.

b) Show that the arclength is less than 1.2 by estimating the integral.

(Indicate how you are estimating it. It helps to write the integral in the simplest-looking form.)

~~**Problem 5.** (15)~~

~~A circle in the xy -plane has radius a and center at the point $(b, 0)$ on the x -axis; assume $a < b$.~~

~~a) Write its equation in rectangular coordinates, and change the equation to polar coordinates.~~

~~Problem 5b can be skipped; no question similar to 5b will appear on the 2006 exam.~~

~~b) Check your answer by deriving it directly in polar coordinates, using a geometric formula.~~

Problem 6. (20)

Once every century the Klingon moon Bosok completely eclipses the moon Yan-ki. As seen from the city Mitek, the two moons appear as circular discs, with radii respectively $\sqrt{2}$ and 1 (units: dorks).

When the center of Bosok is exactly on the circumference of Yan-ki, the problem is to find the area (in square dorks) of the region of the Yan-ki disc that is not yet covered.

~~Draw a diagram in polar coordinates, placing the center of Bosok at the origin, and the other center on the positive x -axis. Set up a definite integral representing the area of the desired region. Indicate how the significant limit on the integral was determined.~~

b) Evaluate the integral.

(If you have time, check your answer by elementary geometry, but no credit for this, just a warm glow of satisfaction and a smug feeling of superiority.)