

18.01 Practice Exam 3

Problem 1.

- a) (10) Derive the trigonometric formula $\cos 2x = 1 - 2\sin^2 x$ and use it to evaluate $\int \sin^2 x dx$.
- b) (10) Differentiate $x \ln x$, and use your answer to evaluate $\int_1^e \ln x dx$.

Problem 2. (15) K-mart is selling at half-price its left-over Great Pumpkins— thin orange plastic shells filled with half-price Halloween candy.

A Great Pumpkin has the shape of the curve $x^2 + y^4 = 1$, rotated about the vertical axis, i.e., the y -axis. This curve is symmetric about the x -axis and the y -axis — it looks something like a circle, but somewhat flatter at the top and bottom.

Using units in feet, how many cubic feet of candy will it take to fill a Great Pumpkin? Give the exact answer, then tell if 5 cubic feet will be enough.

Problem 3. (20: 3,7,5,5) The function $F(x) = \int_0^x t^2 e^{-t^2} dt$ is not elementary; it comes up in calculating the standard deviation of The Curve of normal distribution. (In the following, (a) and (b) go together, but (c) and (d) are both independent questions.)

- a) Find $F'(x)$.
- b) Find the critical point(s) of $F(x)$, and determine their type(s) by studying the sign of $F'(x)$ when x is near a critical point.
- c) Express $\int_0^9 \sqrt{u} e^{-u} du$ in terms of values of $F(x)$.
- d) Estimate $F(x)$ by showing that $F(x) \leq \frac{x^3}{3}$, if $x > 0$.

Problem 4. (15: 12, 3) The end portion of a boneless AllSoy SmartHam of length a has approximately the shape of the region under the curve $y = \sqrt{x}$, $0 \leq x \leq a$, rotated about the x -axis.

- a) When it is sliced vertically into thin slices, what is the average area of a slice?
- b) Where on the SmartHam is there a slice having this average area (i.e., how far from the tip)?

Problem 5. (15: 7,8) You only have time to look at the newspaper on Sunday, but the first thing you turn to is the baseball statistics from Saturday's game, to see how many hits your favorite ball-player Pepe LeMoko got. In September (which started on a Saturday) he had a slump in the middle, but came out of it. His record on the five successive Saturdays was

Day:	1	8	15	22	29
No. hits:	3	2	0	1	3

Suppose there was a game every day; estimate the total number of hits he got during those 29 games by using

- a) the trapezoidal rule
- b) Simpson's rule

Problem 6. (15) Rain falls for 10 hours on a little garden pool, increasing from a drizzle to a downpour, then tapering off to a drizzle again. The rate of raining is given by

$$r(t) = t^2(10 - t)^2 \text{ cm/hr.}$$

This being the city, the rain is polluted with acid; at the start of the rain ($t = 0$), it contains 2 nanograms/cu.cm. of acid, but this decreases linearly to only 1 nanogram/cu.cm. by the end of the rain.

Assume the pool has an area of one square meter.

Set up, but **do not evaluate** a definite integral which tells how many nanograms of acid are in the pool at the end of the rain. Give brief reasoning, either by dividing up the time interval into small subintervals, or by using infinitesimal time intervals.