

### 18.01 Practice Questions for Exam 3 – Fall 2006

1. Evaluate     a)  $\int_0^1 \frac{x dx}{\sqrt{1+3x^2}}$      b)  $\int_{\pi/3}^{\pi/2} \cos^3 x \sin 2x dx$

2. Evaluate  $\int_0^1 x dx$  directly from its definition as the limit of a sum.

Use upper sums (circumscribed rectangles). You can use the formula  $\sum_1^n i = \frac{1}{2}n(n+1)$ .

3. A bank gives interest at the rate  $r$ , compounded continuously, so that an amount  $A_0$  deposited grows after  $t$  years to an amount  $A(t) = A_0 e^{rt}$ .

You make a daily deposit at the constant annual rate  $k$ ; in other words, over the time period  $\Delta t$  you deposit  $k\Delta t$  dollars. Set up a definite integral (give reasoning) which tells how much is in your account at the end of one year. (Do not evaluate the integral.)

4. Consider the function defined by  $F(x) = \int_0^x \sqrt{3 + \sin t} dt$ . Without attempting to find an explicit formula for  $F(x)$ ,

a) (5) show that  $F(1) \leq 2$ ;

b) (5) determine whether  $F(x)$  is convex (“concave up”) or concave (“concave down”) on the interval  $0 < x < 1$ ; show work or give reasoning;

c) (10) give in terms of values of  $F(x)$  the value of  $\int_1^2 \sqrt{3 + \sin 2t} dt$ .

5. If  $\int_0^x f(t) dt = e^{2x} \cos x + c$ , find the value of the constant  $c$  and the function  $f(t)$ .

6. A glass vase has the shape of the solid obtained by rotating about the  $y$ -axis the area in the first quadrant lying over the  $x$ -interval  $[0, a]$  and under the graph of  $y = \sqrt{x}$ . By slicing it horizontally, determine how much glass it contains.

7. A right circular cone has height 5 and base radius 1; it is over-filled with ice cream, in the usual way. Place the cone so its vertex is at the origin, and its axis lies along the positive  $y$ -axis, and take the cross-section containing the  $x$ -axis. The top of this cross-section is a piece of the parabola  $y = 6 - x^2$ . (The whole filled ice-cream cone is gotten by rotating this cross-section about the  $y$ -axis.)

What is the volume of the ice cream? (Suggestion: use cylindrical shells.)

8. Rectangles are inscribed as shown in the quarter-circle of radius  $a$ , with the point  $x$  being chosen randomly on the interval  $[0, a]$ . Find the average value of their area.

9. Find the approximate value given for the integral below by the trapezoidal rule and also by Simpson’s rule, taking  $n = 2$  (i.e., dividing the interval of integration into two equal subintervals):

$$\int_0^{\pi/2} \sin^6 x dx$$

Other possible problems: Volumes by vertical slicing 4B, Work problems (P.Set 5)