MATH 18.01 Problem Set 8 - Spring 2009

Due Thursday, Apr. 16 at 1:00

Part I (10 points)

Lecture 22. (*Thurs., Apr. 2*) Review for Midterm II.

Lecture 23. (*Fri., Apr. 3*) Midterm II (in class).

Lecture 24. (*Tues., Apr. 7*) Inverse trigonometric functions, reduction formulas.
Read: Simmons 9.5, 10.3 *Work:* 5A-2, 5B-13, 16

Lecture 25. (*Thurs., Apr. 9*) Trigonometric integral formulas, trigonometric substitution.
Read: Simmons 10.3, 10.4 *Work:* 5C-2, 4, 5, 6, 5D-5

Lecture 26. (*Fri., Apr. 10*) Completing the square.
Read: Simmons 10.5 *Work:* 5D-10, 11, 13

Lecture 27. (*Tues., Apr. 14*) Partial fractions.
Read: Simmons 10.3, 10.4, Notes F *Work:* 5E-1, 2, 6, 10ae

Part II (15 points)

Problem 1. (*8 pts: 1+2+2+1+1+1*) Consider the function $f(x) = \dfrac{1}{4 - x^2}$. In this problem you will calculate anti-derivatives and integrals of $f(x)$ in a number of ways.

a) Find the partial fraction decomposition of $f(x)$.

b) Use part a) to find an anti-derivative of $f(x)$.

c) Use a trigonometric substitution to find an anti-derivative of $f(x)$.

d) Explain the relationship between your answers in parts a) and c).

e) Calculate the integral $\displaystyle\int_{x=0}^{1} f(x)\,dx$.

f) Try to evaluate the integral $\displaystyle\int_{x=1}^{3} f(x)\,dx$. Does your answer make any sense geometrically? Sketch the graph of $f(x)$ and explain your reasoning in terms of areas under curves.

Problem 2. (*3 pts*) Evaluate the integral

$$\int_{x=0}^{2} \frac{1}{x^4 + 8x^2 + 16}\,dx.$$

Hint: Factor the denominator and use a trigonometric substitution.

Problem 3. (*4 pts: 1+3*) *Constrained growth* models occur very frequently in the study of populations that live in environments with a fixed, limited amount of natural resources. The population initially grows slowly when it is small, then begins to flourish and grow more rapidly, but eventually reaches a point of diminishing returns where it is difficult to achieve significant additional growth. The corresponding differential equation has the form

$$\frac{dP}{dt} = P \cdot (N - P),$$

where $P(t)$ is the population at time t, and N represents the maximum population that can be supported by the environment.

a) At what population is the growth rate maximized?

b) Solve the differential equation subject to the initial condition $P(0) = n$. Sketch your solution and verify that it does describe "constrained" growth.