

## MATH 18.01 Problem Set 7 - Spring 2009

Due Thursday, Apr. 2 at 1:00

### Part I (10 points)

**Lecture 19.** (*Thurs., Mar. 19*) Arc length, surfaces of revolution.

*Read:* Simmons 7.5, 7.6

*Work:* 4F-1abd, 2, 4G-1, 2, 3

**Lecture 20.** (*Fri., Mar. 20*) Work.

*Read:* Simmons 7.7

*Work:* 4J-1, 5

**Lecture 21.** (*Tues., Mar. 31*) Substitution, Basic trigonometric integrals.

*Read:* Simmons 10.1, 10.2, 10.4, 9.3

*Work:* 5D-1, 2, 3, 5

### Part II (15 points)

**Problem 1.** (*8 pts: 2+2+2+2*) The curve  $x^{2/3} + y^{2/3} = 1$  is known as a hypocycloid with 4 cusps.

a) Sketch the curve. Note that the cube root takes precedence over the square in the exponent, so  $(-1)^{2/3} = ((-1)^{1/3})^2 = (-1)^2 = 1$ . This means that  $x^{2/3}$  is always positive. As a further hint, observe that the points  $(\pm 1, 0)$  and  $(0, \pm 1)$  are on the curve.

b) Use implicit differentiation to calculate the derivative  $\frac{dy}{dx}$ .

c) Simplify  $1 + \left(\frac{dy}{dx}\right)^2$  using part b) and the defining equation for the curve.

d) Calculate the total arc-length of the hypocycloid.

**Problem 2.** (*4 pts: 2+2*) Two cylindrical tanks of radius 1m and depth 5m are full of water (density  $1000\text{kg/m}^3$ ) and must be drained.

a) In the first tank, the water is pumped through an inflexible tube that, once inserted, reaches all the way to the bottom. How much work is done in emptying the tank?

b) In the second tank, a flexible, buoyant hose floats on the top of the water as it drains. How much total work is done in emptying the tank?

**Problem 3.** (*3 pts: 2+1*) a) Verify that  $F(x) = -2\cos(2x)$  and  $G(x) = 4\sin^2(x)$  are both anti-derivatives of the same function.

b) What is the relation between  $F(x)$  and  $G(x)$ ?