MATH 18.01 Problem Set 4 - Spring 2009

Due Thursday, Mar. 5 at 1:00

Part I (10 points)

Lecture 10. (*Thurs., Feb. 26*) Mean value theorem, Exam review.

Read: Simmons 2.6 *Work:* 2G-1, 4, 6

Lecture 11. (*Fri., Feb. 27*) Exam 1.

Lecture 12. (*Tues., Mar. 3*) Areas under curves, definite integrals, Riemann integrals.

Read: Simmons 6.1, 6.2, 6.4, 6.5 *Work:* 3B-3

Part II (15 points)

Try each problem alone for 15 minutes before collaborating, and write up solutions independently. The problems are given in order according to the lecture schedule above.

Problem 1. (*5 pts: 2+1+2*) Suppose that $f(x)$ is a differentiable function such that

$$f(0) = 1 \quad \text{and} \quad f'(x) \geq 1 \text{ for all } x.$$

a) Show that $f(x) \geq x + 1$ for $x \geq 0$.

b) If $f(2) = 3$, what can you conclude about $f(x)$ in the range $0 \leq x \leq 2$?

c) Is it possible that $f(-2) = 0$?

Problem 2. (*5 pts: 2+2+1*) A retail store has paid expensive consultants to observe shopper patterns. The store is open from 10:00 A.M. to 8:00 P.M., and the report showed that the rate of hourly sales (#units/hour) behaves as the function $S(t) = 100 - 4(t-5)^2$, where t is the number of hours after opening.

a) Approximate the total daily sales by hourly *initial* segments: the sales for the entire hour $t \in [n, n+1]$ are assumed to be at the same rate as $S(n)$. (*Note: The exact amount is* 667 *units sold*).

b) Approximate the total daily sales by hourly *final* segments: the hour $t \in [n, n+1]$ is assumed to be the same as $S(n+1)$.

c) Compare your answers from parts a) and b). Explain the result geometrically by looking at Riemann rectangles on the graph of $S(t)$, noting that we have now related the total sales to the area under the sales rate!

Problem 3. (*5 pts: 2+2+1*) In this problem you will calculate the area under the line $y = mx$ over the range $a \leq x \leq b$.

a) Write down a Riemann sum with n rectangles that approximates the area.

b) Evaluate the sum from a) using the fact that $1 + 2 + \cdots + n = \dfrac{(n+1)n}{2}$.

c) Find the area of the trapezoid using basic geometry and check that your answer in b) is consistent as $n \to \infty$.