

MATH 18.01 Problem Set 2 - Spring 2009

Due Thursday, Feb. 19 at 1:00

Part I (10 points)

Lecture 5. (*Thurs., Feb. 12*) Exponential/trigonometric derivatives.

Read: Simmons 3.4, 8.1 - 8.3, 9.1, 9.2, 9.4 *Work:* 1I-1abcefkm, 1J-1abdeginp

Additional practice: 1H-2, 3ab, 5

Lecture 6. (*Fri., Feb. 13*) Implicit differentiation, inverse functions, higher derivatives.

Read: Simmons 3.5, 3.6

Work: 1F-2, 3, 4, 8ac, 1G-1ab

Part II (15 points)

Try each problem alone for 15 minutes before collaborating, and write up solutions independently. The problems are given in order according to the lecture schedule above.

Problem 1. (*4 pts: 1+1+1+1*) At the beginning of retirement, a worker withdraws his life-savings of \$1,000,000 to hold in cash. With a steady inflation rate of 10%, this money has *real* value $1,000,000e^{-.1t}$ after t years. As part of his pension, he also receives an inflation-indexed payment of \$10,000 each year.

a) Sketch the graph of his total real wealth as a function of time,

$$f(t) = 1,000,000e^{-.1t} + 10,000t.$$

b) What is the average rate of growth of his wealth during his first 10 years of retirement?

c) In which year is his real wealth declining most quickly?

d) In which year will his wealth be at a minimum?

Problem 2. (*4 pts: 1+1+1+1*) Calculate the 101-st derivative of the following functions:

a) $f(x) = x^{101}$

c) $f(x) = e^{-x}$

b) $f(x) = x^{102}$

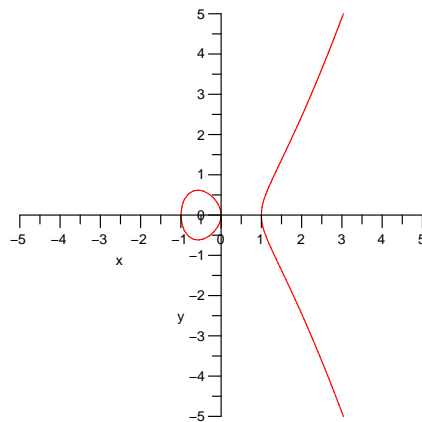
d) $f(x) = \cos 2x.$

Problem 3. (*2 pts*) Compute the derivative of both sides of the double angle formula for cosine,

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta.$$

Can you identify the resulting identity?

Problem 4. (*3 pts*) The equation $y^2 = x^3 - x$ is an example of an *elliptic curve* (which are very important in number theory and cryptography). Find all points on the curve where the tangent line has slope 1. The graph to the right should give you a hint for the number of solutions!



Problem 5. (*2 pts*) Use the rule for derivatives of inverse functions to calculate the derivative of $\arccos(x)$.