MATH 18.01 Problem Set 10 - Spring 2009

Due Thursday, Apr. 30 at 1:00

Part I (10 points)

Lecture 30. (*Thurs., Apr. 23*) Infinite series, convergence.
Read: Simmons 13.1, 13.2 *Work:* 7A-1abcde, 4a, 5
Lecture 31. (*Fri., Apr. 24*) Divergent series, comparison tests.
Read: Simmons 13.3, 13.4, 13.5 *Work:* 7A-3, 7B-2abcdf
Lecture 32. (*Tues., Apr. 28*) Integral test, alternating series.
Read: Simmons 13.6, 13.8 *Work:* 7A-1f, 7B-1abcf, B-4egh

Part II (15 points)

Problem 1. (*8 pts: 3+2+3*) In this problem you will consider the infinite series

$$\sum_{n=1}^{\infty} \frac{1}{n(n+1)\cdots(n+k)}$$

for any $k \geq 1$.

a) Calculate the partial fraction decompositions of $\frac{1}{n(n+1)}$ and $\frac{1}{n(n+1)(n+2)}$. Try to make an educated guess at the decomposition for a general k (recall Pascal's triangle!).

b) Use a comparison to determine whether the sums converge or not for any given k.

c) Use part a) and telescoping to evaluate the sums for $k = 1$ and $k = 2$. Guess at the formula for general k.

Problem 2. (*7 pts: 1+2+2+2*) *The Fly Problem.* Two trains are approaching each other on the same track, each traveling at 50mph. When they are exactly 100 miles apart, a fly begins traveling back and forth between them at 100mph, changing direction each time he hits one of the trains.

a) Calculate the time before the fly first turns around. How far did the fly travel during this time? How much distance remains between the two trains at this time?

b) Calculate the time between the fly's first direction change from part a) and the next direction change. How far does the fly travel during this time period?

c) Identify a geometric series and evaluate to find the total distance traveled by the fly before the trains meet.

d) Check your answer by doing the problem the "easy way": calculate the total time before the trains collide, and use the fly's constant velocity of 100mph to find the total distance traveled.

Remark. The approach in part d) also allows you to easily calculate the distance traveled by the fly even if the two trains are moving at different speeds (for example, 60 and 40mph); the geometric series in this case becomes significantly more complicated.