

## MATH 18.01 Problem Set 1 - Spring 2009

Due Thursday, Feb. 12 at 1:00

### Part I (10 points)

**Lecture 1.** (*Tues., Feb. 3*) Graphing, Limits (in recitation)

*Read:* Notes G, C

*Work:* 1A-1, 2, 9ac, 1D-1bdei

**Lecture 2.** (*Thurs., Feb. 5*) Continuity, Tangents, Derivatives.

*Read:* Simmons 2.1 – 2.3

*Work:* 1D-2abde, 5, 1C-3abd, 4ad, 6

**Lecture 3.** (*Fri., Feb. 6*) Velocity, Trigonometric functions and limits.

*Read:* Simmons 2.4, 2.5

*Work:* 1B-1, 1D-8b, 9

**Lecture 4.** (*Tues., Feb. 10*) Differentiation rules, Chain rule.

*Read:* Simmons 3.1 - 3.3

*Work:* 1E-1, 5abd, 1F-1, 7abd

**Lecture 5.** (*Thurs., Feb. 12*) Exponential/trigonometric derivatives.

*Read:* Simmons 3.4, 8.1 - 8.3, 9.1, 9.2, 9.4

*Work:* (PSet 2)

### Part II (15 points)

Try each problem alone for 15 minutes before collaborating, and write up solutions independently. The problems are given in order according to the lecture schedule above.

**Problem 1.** (*5 pts: 1+1+1+1+1*) A function is *even* if  $f(-x) = f(x)$ , and it is *odd* if  $f(-x) = -f(x)$ .

a) Determine whether the functions  $f(x) = x^4 + 2$ ,  $f(x) = x^3 \sin(x)$ , and  $f(x) = \frac{1}{(x+1)^2} - \frac{1}{(x-1)^2}$  are even or odd.

b) Is

$$f(x) = \frac{xe^x}{x+1} + \frac{(x+1)e^{-x}}{x}$$

even, odd, or neither?

c) Show that the product of two odd functions is even.

d) It is easy to rewrite  $f(x)$  algebraically as

$$f(x) = \frac{f(x) + f(-x)}{2} + \frac{f(x) - f(-x)}{2}.$$

Show that the first term is always an even function, and the second term is always an odd function.

e) Use part d) to decompose  $f(x) = \frac{x}{x+1}$  as the sum of an even function and an odd function.

**Problem 2.** (*3 pts*) (Simmons 2.2.9) Prove that there is no line passing through the point  $(1, -2)$  that is tangent to the curve  $y = x^2 - 4$ .

**Problem 3.** (*4 pts: 2+2*) The graph of the function  $y = f(x) = \sqrt{a^2 - x^2}$  is a semicircle of radius  $a$  centered at the origin.

- a) For any fixed  $0 \leq x_0 \leq a$ , find the equation of the tangent line of the function at the point  $(x_0, f(x_0))$ .
- b) Consider the line that passes through the points  $(0, 3)$  and  $(2, 0)$ . Find the largest circle centered at the origin that does not intersect this line (the circle may be tangent to the line).

**Problem 4.** (4 pts: 2+2) Suppose that  $f(x)$  and  $g(x)$  are differentiable functions.

- a) Calculate  $\frac{d}{dx}(f(x)g(x))$ ,  $\frac{d^2}{dx^2}(f(x)g(x))$ , and  $\frac{d^3}{dx^3}(f(x)g(x))$ .
- b) Expand and simplify  $(a + b)$ ,  $(a + b)^2$ , and  $(a + b)^3$  and compare to part a). Conjecture a general formula for the higher derivatives of  $f(x)g(x)$ .