

18.01 Exam 3 Review Problems

1. Evaluate the integrals:

(a) $\int \cos^5 x \, dx$

(b) $\int \tan^3 x \, dx$

(c) $\int \sin^2 x \cos^5 x \, dx$

(d) $\int_{x=0}^{\pi} \cot^2 x \, dx$

(e) $\int \sin^3 3x \cos^2 3x \, dx.$

2. Use trigonometric substitutions to calculate the following integrals.

(a) $\int \frac{x}{(a^2 - x^2)^{5/2}} \, dx$

(b) $\int \frac{x^3}{4 + x^2} \, dx$

(c) $\int_{x=0}^1 x\sqrt{1-x^2} \, dx$

(d) $\int \frac{9 + x^2}{x} \, dx$

(e) $\int \frac{1}{x^3\sqrt{x^2 - 16}} \, dx.$

3. Evaluate the following integrals by completing the square.

(a) $\int \frac{1}{x^2 - x + 1} \, dx$

(b) $\int_{x=0}^1 \frac{x + 3}{5 + 4x - x^2} \, dx$

(c) $\int \frac{1}{(x^2 + 2x + 3)^{3/2}} \, dx.$

4. Use partial fractions to evaluate the following integrals.

(a) $\int \frac{x^4 - 7x + 1}{x^3 - x} \, dx$

(b) $\int \frac{x + 3}{x^3 + 2x + 2} \, dx$

(c) $\int \frac{1}{(x - 1)^2(x^2 + 2)} \, dx.$

(d) $\int \frac{x^5}{(x^2 + 4)^2} \, dx.$

5. Evaluate the following integrals using integration by parts.

(a) $\int x^2 \ln(x + 1)^2 \, dx$

(b) $\int x \arcsin x \, dx$

(c) $\int x \sec^2 x \, dx.$

(d) $\int_{x=0}^{\pi/2} x^2 \sin x \, dx.$

6. Determine whether each integral converges, and try to evaluate those that do.

$$(a) \int_{x=3}^{\infty} e^{-x} dx$$

$$(b) \int_{x=-1}^{\infty} \frac{1}{(x+2)(x+3)} dx$$

$$(c) \int_{x=1}^{\infty} e^{-x} \cos\left(\frac{1}{x}\right) dx$$

$$(d) \int_{x=-2}^{-1} \frac{x}{(x+2)(x+3)} dx$$

$$(e) \int_{x=1}^2 \frac{1}{\sqrt{x}} + \frac{1}{\sqrt{x+2}} dx$$

$$(f) \int_{x=2}^{\infty} \frac{x}{x^4+2} dx.$$

7. Determine whether each series converges.

$$(a) \sum_{n=1}^{\infty} \sin(n\pi)$$

$$(b) \sum_{n=1}^{\infty} \sin\left(\frac{(2n+1)\pi}{2}\right)$$

$$(c) \frac{1}{2} + \frac{1}{8} + \frac{1}{32} + \frac{1}{128} + \dots$$

$$(d) \sum_{n=1}^{\infty} \frac{2}{n(n+2)}$$

$$(e) \sum_{n=1}^{\infty} \frac{n}{3n-2}$$

$$(f) \sum_{n=1}^{\infty} \frac{1}{(3n+1)^2}$$

$$(g) \frac{1}{2} - \frac{1}{4} + \frac{1}{6} - \frac{1}{8} + \dots$$

$$(h) \sum_{n=1}^{\infty} \frac{3n}{\sqrt{n^3 - 5n^2 + 5}}$$

$$(i) \sum_{n=1}^{\infty} \frac{n}{e^n}$$

$$(j) \sum_{n=1}^{\infty} \frac{(\ln n)^2}{n}.$$