18.01 Exam 2 Review Problems

1. Calculate the integrals

   (a) $\displaystyle\int_2^5 3\,dx$,

   (b) $\displaystyle\int_0^{3\pi/2} \cos(x)\,dx$

   (c) $\displaystyle\int_1^3 x^3 - 5x\,dx$,

   (d) $\displaystyle\int_a^b \frac{d}{dx} f(x)\,dx$,

   (e) $\displaystyle\int_1^3 x\sqrt{2x^2+1}\,dx$

   (f) $\displaystyle\int_0^{1000} \frac{x^2}{x^3+1}\,dx$,

   (g) $\displaystyle\int_{-1}^1 x^3 \sin^2(x-1)\,dx$,

   (h) $\displaystyle\int_0^{1/2} e^{2x}\,dx$.

   Extra practice: evaluate as many of these integrals as you can using the Riemann sum definition.

2. Find anti-derivatives for the functions

   (a) $f(x) = x^3 - x$,

   (b) $f(x) = \sin(x) - \sec^2(x)$,

   (c) $f(x) = e^{2x}$,

   (d) $f(x) = \dfrac{1}{\sqrt{1-x^2}}$,

   (e) $f(x) = \dfrac{1}{1+x^2}\, e^{\tan(x)}$,

   (f) $f(x) = \dfrac{e^x}{1+e^x}$,

   (g) $f(x) = 3^x$,

   (h) $f(x) = \dfrac{x}{(x^2+2)^2}$.

3. (a) Find a function that satisfies $f'(x) = \cos(x)$ and $f(0) = 1$.

   (b) Calculate the area under the function $f(x) = x^3 + e^{-x}$ from $x = 0$ to 4.

   (c) Calculate the area between the graphs of $y = 2x^2$ and $y = 1 + x^4$.

   (d) Calculate the area between the functions $y = x$ and $y = \cos(x)$.

4. Compute the derivatives of the integrals

   (a) $\displaystyle\frac{d}{dt}\int_2^t (x+1)^4\,dx$,

   (b) $\displaystyle\frac{d}{dt}\int_2^{e^t} \sin(x)\,dx$,

   (c) $\displaystyle\frac{d}{dt}\int_2^6 e^{\sqrt{x-5}}\,dx$,

5. Find the solution to the differential equation $\dfrac{dy}{dx} = y^2 e^x$ that passes through the point $(0, 2)$.

6. (a) Calculate the arc-length of $y = \dfrac{x^3}{6} + \dfrac{1}{2x}$ from $x = 2$ to 3.

(b) Write down an integral that calculates the arc-length of $y = e^{x^2}$ from $x = 0$ to 1.

(c) Calculate the area of the surface of revolution for the curve $y = x^3$ from $x = 1$ to 3 rotated about the x-axis.

(d) Compute the surface area of revolution of the curve $y = x^2$ from $x = 0$ to 2 rotated about the y-axis.

7. For each of the following functions, use both the shell and disk methods to write down integrals that calculate the volumes of revolution for the range and axes indicated. Evaluate the integrals when possible.

(a) $y = 1 + x,\ 1 \leq x \leq 3,\ x$-axis,

(b) $y = 1 + x^2,\ 1 \leq x \leq 3,\ y$-axis,

(c) $y = 1 + x,\ 1 \leq x \leq 3,\ x$-axis,

(d) $y = \dfrac{1}{x^2},\ 1 \leq x \leq 100,\ x$-axis.

(e) The portion of the first quadrant between $y = \sqrt[3]{x}$ and $y = x$ around the y-axis,

8. Gravity repels two masses m_1 and m_2 with force Gm_1m_2/r^2, where G is the gravitational constant and r is the distance between the two. Suppose that a 1kg mass is placed 100m from a 20kg mass. How much work is required to move the first mass so that it is 5m from the second?

9. A cylindrical tank with radius 10ft and height 20ft has a cylindrical water pump resting on its base; the pump has radius 3ft and height 5ft. If the density of water is δ, how much work is required to fill the tank with water? Note that there is less water in the bottom of the tank due to the presence of the pump.