

18.01 Exam 1 Review Problems

1. Calculate the derivatives of

(a) $f(x) = e^{x^2}$,

(d) $f(x) = x^{x^2}$,

(b) $f(x) = \frac{x^2 - 1}{x^2 + 1}$.

(e) $f(x) = \arctan x^2 - 1$,

(c) $f(x) = \frac{\sin x}{\sqrt{x}}$,

(f) $f(x) = 2^{x^x}$,

2. Identify critical points, inflection points, and sketch the graphs of

(a) $f(x) = x^3 - x$,

(d) $f(x) = \sin x + \cos x$.

(b) $f(x) = \frac{e^x - 1}{x}$,

(e) $f(x) = x \ln(x^2)$.

(c) $f(x) = x \cos x$.

(f) $f(x) = x^4 - 2x^2 + 1$.

3. A 3-inch cubic icecube left in the sun is melting at the rate of $1 \text{ in}^3/\text{min}$. The melting occurs evenly on all sides, so it maintains a cubic shape. How quickly is the side-length of the cube shrinking when it has shrunk to a 1-inch cube?

4. Consider the implicitly defined curve $x^2 - y^2 = 1$. Find all points where the tangent slope is 0 or 1. Write down a general equation for *all* tangent lines. Sketch the curve as best you can, using any techniques.

5. It costs a pizza shop \$3 in labor to bake any pizza, and the ingredients add an additional cost of 3 cents per square inch. If they can sell a pizza at \$1 for each inch of radius, what is the most profitable size?

6. A car traveling east at 50m.p.h. passes below a flock of birds 3 miles overhead that are traveling south at 20m.p.h. How quickly is the distance between them growing after 1 hour has passed?

7. Evaluate the limits

(a) $\lim_{x \rightarrow 0^+} \tan x$.

(e) $\lim_{x \rightarrow \infty} x^2 + \ln(1/x)$.

(b) $\lim_{x \rightarrow 0} \frac{\tan x}{x}$.

(f) $\lim_{x \rightarrow 0} \left(\frac{1}{x}\right)^x$.

(c) $\lim_{x \rightarrow 0} \frac{e^{x^2} - 1}{x^3}$.

(g) $\lim_{x \rightarrow \infty} \left(\frac{1}{x}\right)^x$.

(d) $\lim_{x \rightarrow 1} (x^2 - 1) \ln x$.