Quick facts

Anatomy	rectangular	polar	notes
complex number	$a+ib$	$Ae^{i\theta}$	a,b,θ,A real and $A\geq 0$
sinusoid	$acos(\omega t)+bsin(\omega t)$	$Acos(\omega t-\theta)$	a,b,θ,A same as above
	$\mathcal{R}e\{(a+ib)e^{-i\omega t}\}$	$\mathcal{R}e\{Ae^{i\theta}e^{-i\omega t}\}$	a,b,θ,A consistent as above

complex vocab: A is magnitude, modulus, absolute value, or amplitude. θ is phase, angle, argument.
sinusoid vocab: A is amplitude. θ is phase lag or phase shift.

Solutions	equation	solution	notes
1^{st} order LTI	$(\hat{D}-r_1\hat{I})x(t)=e^{r_2t}$	$x(t)=\frac{1}{r_2-r_1}e^{r_2t}+Ce^{r_1t}$	only if $r_1\neq r_2$
	$\hat{D}-r\hat{I})x(t)=e^{rt}$	$x(t)=te^{rt}+Ce^{rt}$	
2^{nd} order	$p(\hat{D})x_h(t)=0$		
homogeneous	$(\hat{D}^2+b\hat{D}+k\hat{I})x_h(t)=0$		
(overdamped)	$(\hat{D}-r_1\hat{I})(\hat{D}-r_2\hat{I})x_h(t)=0$	$x_h(t)=C_1e^{r_1t}+C_2e^{r_2t}$	if $r_1\neq r_2$ real
(critical)		$x_h(t)=C_1te^{rt}+C_2e^{rt}$	if $r_1=r_2=r$
(underdamped)		$x_h(t)=e^{-at}(C_1cos(\omega t)+C_2sin(\omega t))$	if $r_1,r_2=-a\pm i\omega$
2^{nd} order	$p(\hat{D})x_p(t)=e^{st}$		
particular	$(\hat{D}^2+b\hat{D}+k\hat{I})x_p(t)=e^{st}$	$x_p(t)=\frac{1}{p(s)}e^{st}$	only if $p(s)\neq 0$
2^{nd} order			
general	$p(\hat{D})x_p(t)=e^{st}$	$x(t)=x_p(t)+x_h(t)$	

Linear ODE vocab: $x_p(t)$ is the particular, periodic, or (sometimes) sinusoidal, solution. $x_h(t)$ is the homogeneous, transient, autonymous, complementary, solution.

Quadratic	roots
overdamped	$r_1,r_2=-b/2\pm\sqrt{(b/2)^2-k}$
critically damped	$r_1=r_2=-b/2$
underdamped	$r_1,r_2=a\pm i\omega$
	$a=-b/2$ and $\omega=\sqrt{k-(b/2)^2}$

Generic homogeneous: If all the roots of $p(r)$ are not equal,

$$\begin{aligned} p(\hat{D})x_h(t) &= 0 \\ x_h(t) &= C_1e^{r_1t}+C_2e^{r_2t}+\ldots+C_ne^{r_nt} \end{aligned}$$

Exponential response: If $p(s)\neq 0$,

$$\begin{aligned} p(\hat{D})x_p(t) &= e^{st} \\ x_p(t) &= \frac{1}{p(s)}e^{st} \end{aligned}$$

Undetermined coefficients: If $p(0)\neq 0$,

$$\begin{aligned} p(\hat{D})x_p(t) &= q_nt^n+q_{n-1}t^{n-1}+\ldots+q_1t+q_0 \\ x_p(t) &= x_nt^n+x_{n-1}t^{n-1}+\ldots+x_1t+x_0 \end{aligned}$$

plug in and solve for coefficients $x_n,x_{n-1},\ldots,x_1,x_0$.

Exponential shift

$$\begin{aligned} p(\hat{D})x(t) &= e^{st}g(t) \\ p(\hat{D}-s\hat{I})(e^{-st}x(t)) &= g(t) \end{aligned}$$