

18.100A – PRACTICE MIDTERM #1

Not to be handed in.

No books, no notes, no calculators.

Time: 50 Minutes

1.

- (a) Let $\{a_n\}$ be a sequence. Write down the definition of the statement meaning that $\{a_n\}$ converges to L , which we denote by $a \rightarrow L$ or $\lim a_n = L$.
- (b) Prove that $1/\sqrt{n} \rightarrow 0$ directly from the definition of convergence.
- (c) Give an example for a sequence $\{a_n\}$ that *does not* converge. (You don't have to justify your example.)

2.

- (a) Let $\sum a_n$ be a series. Write down the definition of the statement meaning that $\sum a_n$ converges.
- (b) Give an example for a series $\sum a_n$ that *does not* converge. (You don't have to justify your example.)

3. The following statements are either true or false. If true, give a proof. If false, give a counterexample.

- (a) If for any $\epsilon > 0$ we have $a_n \geq 1 - \epsilon$ for $n \gg 1$, then $a_n \rightarrow 1$.
- (b) $a_n \rightarrow 0 \iff |a_n| \rightarrow 0$.
- (c) $a_n \rightarrow L \iff |a_n| \rightarrow |L|$.
- (d) If $\{a_n\}$ converges, then $\{a_n\}$ is bounded.
- (e) The sequence $\{a_n\}$ with $a_n = (-1)^n + \frac{1}{n}$ has no cluster points.
- (f) If $\{a_n\}$ is bounded, then $\{a_n\}$ converges.
- (g) If $|\frac{a_{n+1}}{a_n}| < 1$ for all n , then $\sum a_n$ converges.
- (h) If $a_n > 0$ and $\{a_n\}$ is decreasing, then $\sum (-1)^n a_n$ converges.

Further Preparation for Midterm: Work through the **Question Sections** in Chapters 1 to 7. (You find the answers to these questions at the end of each chapter.)

ENNO LENZMANN, M.I.T., DEPARTMENT OF MATHEMATICS, ROOM 2-230
E-mail address: lenzmann@math.mit.edu