

# HOMework 5

due Wednesday, August 6, at 10:45am in 36-112

Reminder: You are encouraged to work with others, but each student must write up his or her own solutions. List the names of all collaborators on the front page of the problem set.

## Part I

1. [9 pt] Find the following areas.
  - (a) [2 pt] The region below  $e^{x/2}$  between  $x = -\ln 4$  and  $x = \ln 4$ .
  - (b) [4 pt] The region between  $y = 0$ ,  $x = 0$ ,  $x = 1$  and  $f(x) = \sqrt{1 - x^2}$ . What does this imply?
  - (c) [3 pt] The region between  $f(x) = 1 - x^2$  and  $g(x) = x^4 - 1$ .
2. [8 pt] Evaluate the following:
  - (a)  $(2 + 3i)(3 - 4i)$
  - (b)  $(-1 + 5i)^2$
  - (c)  $(4 - i)/(2 + i)$
  - (d)  $|1 + 3i|$
  - (e) [2 pt]  $e^{2+3\pi i}$
  - (f) [2 pt]  $(1 + i)^{365}$

## Part II

3. [7 pt] An ellipse with axes  $a$  and  $b$  is given by the formula  $x^2/a^2 + y^2/b^2 = 1$ . Find its area.
4. [4 pt]
  - (a) [2 pt] Prove that  $\cos^2(x) = (1 + \cos 2x)/2$ .
  - (b) [2 pt] Find the average of  $\cos^2(x)$  between  $n\pi$  and  $m\pi$  for any integers  $n, m$ .
5. [6 pt] Consider the region  $R$  in the *first quadrant* bounded above by  $y = x^2 + 1$ , below by  $y = 2x - 2$ , and to the right by the line  $x = 2$ . Set up an integral (but do not evaluate) for the volume of each of the following solids:
  - (a) [2 pt] the solid obtained by rotating  $R$  about the  $y$ -axis.
  - (b) [2 pt] the solid obtained by rotating  $R$  about the  $x$ -axis.

- (c) [2 pt] the solid obtained by rotating  $R$  about the line  $y = 6$ .
6. [6 pt] Use Euler's formula  $e^{ix} = \cos x + i \sin x$  and  $(e^x)' = e^x$  to prove addition theorems for and find derivatives of  $\cos x$  and  $\sin x$ .