

HOMework 4

due Wednesday, July 30, at 10:45am in 36-112

Reminder: You are encouraged to work with others, but each student must write up his or her own solutions. List the names of all collaborators on the front page of the problem set.

Part I

1. [16 pt] Find the following integrals:

(a) $\int_0^2 (x^3 - x^2 - 3x + 1) dx$

(b) $\int \left(\frac{1}{x} - \frac{1}{2x^3} + 4 \cos 3x - e^{-3x} \right) dx$

(c) $\int x^2 \cos x^3 dx$

(d) $\int_{-\pi/2}^{\pi/2} \frac{\cos x}{1 + \sin^2 x} dx$

(e) $\int \frac{e^x}{\cos^2(e^x)} dx$

(f) $\int (x^2 - 1) \arctan(x^3 - 3x) dx$

(g) [2 pt] $\int_1^{\infty} x \ln x dx$

(h) [2 pt] $\int (x^2 - x) \sin x dx$

(i) [2 pt] $\int \frac{1}{x^2 + 4x + 13} dx$

(j) [2 pt] $\int_{-\infty}^{\infty} \frac{1}{(x^2 + 1)(x^2 + 4)} dx$

(k) [2 pt] $\int \frac{\cos x}{\sin^2 x - \sin x - 6} dx$

2. [6 pt] Solve the following differential equations:

(a) [2 pt] $y' = 6y^{2/3}, y(1) = 8$

(b) [2 pt] $y' = 4x^3(y - 4)^2$

(c) [2 pt] $y' = 3x^2y^2 - y^2, y(0) = 1$

Part II

3. (a) [3 pt] Use induction to prove

$$\sum_{j=1}^n j = \frac{n(n+1)}{2}, \quad \sum_{j=1}^n j^2 = \frac{n(n+1)(2n+1)}{6}.$$

- (b) [3 pt] Use Riemann sums to calculate

$$\int_0^2 (x^2 + 3x - 2) dx.$$

Check your result using the fundamental theorem of calculus.

4. [6 pt] E & P, page 345, problem 48
5. [6 pt] Newton's law of cooling states that

$$\frac{dT}{dt} = -k(T - T_0),$$

where t is time, T is the temperature of the object, T_0 is the surrounding temperature, and k is a constant.

Assume that a corpse is found in a room with constant temperature 20°C . At noon, the temperature of the corpse is 30°C , and at 1pm, the temperature is 25°C . When did the murder take place, assuming that the victim was healthy?