HOMEWORK 1

due Wednesday, July 2, at 10:45am in 36-112

Reminder: You are encouraged to work with others, but each student must write up his or her own solutions. List the names of all collaborators on the front page of the problem set.

Part I

1. [6 pt] Draw the graphs of the following functions.
   (a) $f(x) = 2(x-4)^2 + 3$
   (b) $f(x) = x^5 - 1$
   (c) $f(x) = -2\cos(x + \pi/3)$
   (d) $f(x) = 2\ln(2x)$
   (e) [2 pt] $f(x) = e^{-x}\sin x$

2. [7 pt] Find the following limits. Show work.
   (a) $\lim_{x\to 1} \frac{x^2-x}{x^2+2x-3}$
   (b) $\lim_{x\to\infty} \frac{3x^2-x+3}{6x^3+x+7}$
   (c) $\lim_{x\to-\infty} \frac{4x^3+4x^2-2x-1}{2x^3-x^2+1}$
   (d) $\lim_{x\to 1} \frac{1-x}{|x-1|}$
   (e) $\lim_{x\to 0^-} \lfloor x \rfloor$, where $\lfloor x \rfloor$ is the floor of x
   (f) $\lim_{x\to 0^+} \lfloor x \rfloor$
   (g) $\lim_{x\to\pi} \frac{\tan x}{x-\pi}$

3. [4 pt] Differentiate the following functions.
   (a) $f(x) = x^4 - 2x^2 + 3x - 5$
   (b) $f(x) = \cos x - 2\sin x + 3\tan x - 4\cot x$
   (c) $f(x) = e^x \ln x$
   (d) $f(x) = \cos^2 x$ (use product rule)

Part II

3. [5 pt] By using the definition of the derivative, differentiate $f(x) = x^3$ (at a general point).

4. [5 pt] What is $\frac{d^{100}}{dx^{100}} \ln(1 + 2x)$?

5. [8 pt] Generalize the product rule as follows.

   (a) Let $u(x)$, $v(x)$, and $w(x)$ be differentiable functions. Derive a formula for the derivative of their product uvw.

   (b) Generalize your work in part (a) by finding a formula for the derivative of $u_1 u_2 \cdots u_n$.

   (c) Use your formula from (b) to find the derivative of x^n, where n is a positive integer, without using the power rule.

6. [5 pt] Check that you have to be careful not to reverse the roles of ϵ and δ in the $\epsilon - \delta$ definition of the limit by doing the following. Take the function

$$f(x) = \begin{cases} 0 & : \quad x < 1 \\ 1 & : \quad x \geq 1 \end{cases},$$

which obviously does not have a limit at 1, and prove that for every $\delta > 0$, there exists an $\epsilon > 0$ so that $0 < |x - 1| < \delta$ implies $|f(x) - 1| < \epsilon$. (Hint: ϵ doesn't have to be small!)