Errata for “Good formal structures for flat meromorphic connections, I: Surfaces”

Liang Xiao has pointed out that the proof of Theorem 4.2.3 is incomplete: it assumes that the differential module N of rank 1 over R' is free, and this is not immediate in general (see Remark 3.1.3). The published proof does correctly reduce Theorem 4.2.3 to the fact that for N a rank 1 differential module over $R_{n,m}$, there exists $s \in R_{n,m}$ for which $E(-s) \otimes_{R_{n,m}} N$ is regular. We give here an alternate proof of this statement, by induction on m; the base case $m = 0$ is trivial because N is regular (e.g., by Theorem 4.1.4).

Recall (Notation 4.1.1) that $F_{(m)}$ is defined to be the completion of $\mathrm{Frac}(R_{n,0})$ with respect to the x_m-adic norm. If we put $S = R_{n,m-1}/(x_m)$ and $\kappa_{(m)} = \mathrm{Frac}(S)$, we may identify $F_{(m)} \cong \kappa_{(m)}((x_m))$. Let V_0 be any lattice in $V = N \otimes_{R_{n,m}} F_{(m)}$, and put $N_0 = N \cap V_0$; as in Lemma 4.1.2, the $R_{n,m-1}$-module N_0 is finitely generated and torsion-free.

By Theorem 2.3.3, there exists $s \in F_{(m)}$ for which $E(-s) \otimes_{F_{(m)}} V$ is regular. Suppose that s has x_m-adic valuation $-h < 0$; write $s = \sum_{i=-h}^{\infty} a_i x_m^i$ with $a_i \in \kappa_{(m)}$. Suppose further that $a_{-h} \notin S$; since S is noetherian and factorial, a_{-h} fails to belong to the discrete valuation ring T obtained by localizing S at some minimal nonzero prime ideal. (More concretely, this ideal is generated by an irreducible polynomial dividing the denominator of a_{-h} in lowest terms.) Since $N_0/x_m N_0$ is a nonzero, finitely generated, torsion-free S-module, $(N_0/x_m N_0) \otimes_S T$ is a nonzero finitely generated, torsion-free T-module, and hence a nonzero finite free T-module. In particular, it cannot be stable under multiplication by an element of $\mathrm{Frac}(T)$ not contained in T. However, $(N_0/x_m N_0) \otimes_S T$ is stable under multiplication by a_{-h} because for $\mathbf{v} \in N_0$, we have $g x_m^{h+1} \partial_m(\mathbf{v}) \equiv -h a_{-h} \mathbf{v} \pmod{x_m N_0}$. This contradiction shows that $a_{-h} \in S$.

By lifting a_{-h} to $R_{n,m}$ and replacing N with $E(-a_{-h} x_m^{-h}) \otimes_{R_{n,m}} N$, we may reduce the value of h; by induction, we reduce to the case where V is itself regular. In this case, by Proposition 2.1.12, $x_m \partial_m$ acts on $N_0/x_m N_0$ as multiplication by some scalar belonging to the subfield of $\kappa_{(m)}$ killed by ∂_i for $i \neq m$. This subfield is precisely k, so we may twist once more to force $x_m \partial_m$ to annihilate $N_0/x_m N_0$. In this case, N_0 is a differential module over $R_{n,m-1}$, so by the induction hypothesis we may deduce the claim.

Some additional errata (from Francesco Baldassarri, Andrea Pulita, and Liang Xiao):

- In Proposition 1.6.5, the inclusion $F \subseteq F'$ should be assumed to be isometric. In the proof, “viewed as a differential module over F” should be “viewed as a differential module over F'”.
- In Definition 2.0.1, $N \otimes_R F$ should be $N \otimes_R S$.
- In Definition 2.2.3, “the eigenvalues of $z\partial$ on W” should be “the eigenvalues of $z\partial$ on W/zW”.
- In the proof of Proposition 2.2.8, the third displayed equation holds for $m > 0$ rather than $m \in \mathbb{Z}$.
- In the proof of Proposition 2.2.15(a), it is unnecessary to invoke Lemma 2.1.3(c) because that has already been done at the beginning of the proof.

- In the third paragraph of the proof of Lemma 2.3.1, it is $(e/d)r$ rather than r that should be taken to be a root of $Q(T)$. Then $E(-r) \otimes_{F'} (V \otimes_F F')$ has a cyclic vector with corresponding polynomial $P(T - (e/d)r)$, and so forth.

- In the proof of Proposition 2.5.6, the last displayed equation is missing a factor of $\dim(V)$ on the right side.

- The statement of Lemma 2.6.4 should not assume that $V \otimes_{\mathcal{R}^{\mathrm{bd}}} F$ is regular. The proof gives covers this case, but the general case reduces to this case using [18, Theorem 2.3.9 and Remark 2.3.11]. The general case is what is needed for the applications of Lemma 2.6.4 in the proofs of Proposition 2.7.5 and Theorem 3.3.6. Also, in the proof of Lemma 2.6.4, W should be W_0.

- In the proof of Proposition 2.7.5, one should consider the derivations $\partial_0, \dots, \partial_n$ rather than ∂_0, ∂_1. Similarly, the set $\{0, 1\}$ should be replaced by $\{0, \dots, n\}$ every time it appears.

- In the proof of Theorem 3.2.2, every superscripted d should be replaced by n.

- In Definition 4.3.1, the conditions $\phi_\alpha \notin R_{n,m}$ and $\phi_\alpha - \phi_\beta \notin R_{n,m}$ should read $\phi_\alpha \notin R_{n,0}$ and $\phi - \alpha - \phi_\beta \notin R_{n,0}$, respectively.

- In Corollary 5.3.6 (in both parts), it must also be assumed that $f'_+(\alpha_{\mathbb{D}}, \beta)$ exists for some $\beta \neq \alpha_{\mathbb{D}}$.